Supplemental material to the article
"Phonon Assisted Resonant Tunnelling and its Phonons Control"

Here we present supplementary materials of our model calculations

Oscillatory case. In the case of interaction with two local phonon modes, ζ_n is given by Eq. (13) of our paper. As a result, the sum in the last term of Eq. (4) of our paper can be rewritten as $U = U_1 - U_2$, where

$$U_1 = \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{\Omega_n},\ U_2 = \frac{1}{2}\sum_{n=1}^{\infty}\frac{\cos 2\nu_n\tau_0}{\Omega_n},\quad \Omega_n = \nu_n^2\left(\nu_n^2+\omega_0^2+\nu_n^2\frac{C_2^2}{\omega_2^2(\omega_2^2+\nu_n^2)}+\nu_n^2\frac{C_3^2}{\omega_3^2(\omega_3^2+\nu_n^2)}\right). \tag{1}$$

Let us rewrite U_1 as

$$U_1 = \frac{1}{2}\sum_{n=1}^{\infty}\frac{\omega_2^2\omega_3^2(\omega_2^2+\nu_n^2)(\omega_3^2+\nu_n^2)}{\nu_n^2\omega_2^2\omega_3^2(\nu_n^2-x_1)(\nu_n^2-x_2)(\nu_n^2-x_3)}, \tag{2}$$

where for $S>0$: $x_1 = -2\sqrt{Q}\cos\Phi - A/3,\ x_2 = -2\sqrt{Q}\cos(\Phi+2\pi/3) - A/3,\ x_3 = -2\sqrt{Q}\cos(\Phi-2\pi/3) - A/3$, $Q = (A^2-3B_\omega)/9$; $R = (2A^3 - 9AB_\omega + 27C)/54$; $S = Q^3 - R^2$; $\Phi = \arccos(R/\sqrt{Q^3})/3$, $A = \omega_2^2+\omega_3^2+\omega_0^2+C_2^2/\omega_2^2+C_3^2/\omega_3^2$, $B_\omega = \omega_2^2\omega_3^2+\omega_0^2(\omega_2^2+\omega_3^2)+C_2^2\omega_3^2/\omega_2^2+C_3^2\omega_2^2/\omega_3^2$, $C = \omega_0^2\omega_2^2\omega_3^2$.

The last expression for U_1 can be rewritten in the following form:

$$U_1 = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{\beta_0}{\nu_n^2}+\frac{\gamma}{\nu_n^2-x_1}+\frac{\varphi}{\nu_n^2-x_2}+\frac{\Delta}{\nu_n^2-x_3}\right), \tag{3}$$

where

$$\varphi = \frac{x_2}{x_3(x_2-x_1)}\left\{\Delta\frac{x_2}{x_3}(x_1-x_3) - 1 - \frac{\omega_2^2\omega_3^2}{x_1x_2x_3}(x_2+x_3-x_1) - \frac{x_2+x_3}{x_2x_3}[\omega_2^2+\omega_3^2+\frac{\omega_2^2\omega_3^2}{x_1x_2x_3}(x_1x_2+x_1x_3+x_2x_3)]\right\},$$

$$\beta_0 = -\frac{\omega_2^2\omega_3^2}{x_1x_2x_3},\ \gamma = \frac{1}{x_2x_3}\{\omega_2^2+\omega_3^2-\Delta x_1x_2-\varphi x_1x_3-\beta_0(x_2x_3+x_1(x_2+x_3))\},\ \nu_n = \frac{2\pi n}{\beta},$$

$$\Delta = \frac{x_3^2}{(x_3-x_2)(x_1-x_3)}\left\{\frac{\omega_2^2\omega_3^2}{x_1x_2x_3}\left(\frac{x_1x_2+x_1x_3+x_2x_3}{x_2x_3}-1\right)+\frac{\omega_2^2+\omega_3^2}{x_2x_3}-\right.$$
$$\left.-\frac{1}{x_3}\left(1+\frac{\omega_2^2\omega_3^2}{x_1x_2x_3}\left[\frac{x_1x_2+x_1x_3+x_2x_3}{x_2x_3}+(x_2+x_3-x_1)\right]\right)+\frac{(\omega_2^2+\omega_3^2)(x_2+x_3)}{x_2x_3}\right\}. \tag{4}$$

Using

$$\sum_{n=1}^{\infty}\frac{\beta_0}{\nu_n^2} = \beta_0\sum_{n=1}^{\infty}\frac{\beta^2}{4\pi^2n^2} = \beta_0\frac{\beta^2}{4\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2} = \beta_0\frac{\beta^2}{24},\quad x_1 = -2\sqrt{Q}\cos\phi - \frac{A}{3} = -x_{10} = -(2\sqrt{Q}\cos\phi+\frac{A}{3}), \tag{5}$$

we have for the case $x_1<0$,

$$\sum_{n=1}^{\infty}\gamma(\nu_n^2+x_{10})^{-1} = \sum_{n=1}^{\infty}\gamma(\frac{4\pi^2n^2}{\beta^2}+x_{10})^{-1} = \frac{\gamma\beta^2}{4\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2+\frac{x_{10}\beta^2}{4\pi^2}} = \frac{\gamma\beta^2}{4\pi^2}[-\frac{4\pi^2}{2x_1\beta^2}-\frac{\pi^2}{\sqrt{x_1}\beta}\mathrm{cotan}(\frac{\sqrt{x_1}\beta}{2\pi})]. \tag{6}$$

Denoting

$$\tilde{x}_{10}^2 = \frac{x_{10}\beta^2}{4\pi^2},\quad x_2 = -2\sqrt{Q}\cos(\Phi+\frac{2}{3}\pi)-\frac{A}{3} = -x_{20},\quad \tilde{x}_{20}^2 = \frac{x_{20}\beta^2}{4\pi^2},$$

$$x_3 = -2\sqrt{Q}\cos(\Phi - \frac{2}{3}\pi) - \frac{A}{3} = -x_{30}, \quad \tilde{x}_{30}^2 = \frac{x_{30}\beta^2}{4\pi^2}, \tag{7}$$

we have finally for the case $x_1 > 0, x_2 > 0, x_3 > 0$,

$$U_1 = \frac{1}{2}\{\beta_0\frac{\beta^2}{24} + \frac{\gamma\beta^2}{4\pi^2}[-\frac{4\pi^2}{2x_1\beta^2} - \frac{\pi^2}{\sqrt{x_1}\beta}\mathrm{cotan}(\frac{\sqrt{x_1}\beta}{2})] + \frac{\varphi\beta^2}{4\pi^2}[-\frac{4\pi^2}{2x_2\beta^2} - \frac{\pi^2}{\sqrt{x_2}\beta}\mathrm{cotan}(\frac{\sqrt{x_2}\beta}{2})] +$$
$$+ \frac{\Delta\beta^2}{4\pi^2}[-\frac{4\pi^2}{2x_3\beta^2} - \frac{\pi^2}{\sqrt{x_3}\beta}\mathrm{cotan}(\frac{\sqrt{x_3}\beta}{2})]\}. \tag{8}$$

Let us turn to calculation of U_2.

$$U_2 = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{\beta_0\cos 2\nu_n T_0}{\nu_n^2} + \frac{\gamma\cos 2\nu_n T_0}{\nu_n^2 - x_1} + \frac{\varphi\cos 2\nu_n T_0}{\nu_n^2 - x_2} + \frac{\Delta\cos 2\nu_n T_0}{\nu_n^2 - x_3}\right),$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\frac{\beta_0\cos^2\frac{2\pi T_0}{\beta}n}{\frac{4\pi^2 n^2}{\beta^2}} = \frac{\beta^2\beta_0}{8\pi^2}\sum_{n=1}^{\infty}\frac{\cos^2\frac{2\pi T_0}{\beta}n}{n^2} = \frac{\beta^2\beta_0}{8\pi^2}\frac{1}{12}\left(3\frac{(4\pi T_0)^2}{\beta} - 6\pi\frac{4\pi T_0}{\beta} + 2\pi^2\right), \tag{9}$$

$$\frac{1}{2}\gamma\sum_{n=1}^{\infty}\frac{\cos\frac{4\pi T_0}{\beta}n}{\frac{4\pi^2 n^2}{\beta^2} - x_1} = \frac{\beta^2\gamma}{8\pi^2}\sum_{n=1}^{\infty}\frac{\cos\frac{4\pi T_0}{\beta}n}{n^2 - \frac{x_1\beta^2}{4\pi^2}} = \frac{\beta^2\gamma}{8\pi^2}\left\{\frac{\pi^2}{\sqrt{x_1}\beta}\cos[(\pi - \frac{4\pi T_0}{\beta})\frac{\sqrt{x_1}\beta}{2\pi}]\mathrm{cosec}\frac{\sqrt{x_1}\beta}{2} + \frac{2\pi^2}{x_1\beta^2}\right\}. \tag{10}$$

For the case $x_1, x_2, x_3 > 0$, U_2 then takes the form

$$U_2 = \frac{1}{2}\{\frac{\beta_0\beta^2}{48}(3\frac{(4\pi T_0)^2}{\beta} - \frac{24\pi^2 T_0}{\beta} + 2\pi^2) + \frac{\gamma\beta^2}{4\pi^2}\{\frac{\pi^2}{\sqrt{x_1}\beta}\cos[(\pi - \frac{4\pi T_0}{\beta})\frac{\sqrt{x_1}\beta}{2\pi}]\mathrm{cosec}\frac{\sqrt{x_1}\beta}{2}\} +$$
$$+ \frac{\varphi\beta^2}{4\pi^2}\{\frac{\pi^2}{\sqrt{x_2}\beta}\cos[(\pi - \frac{4\pi T_0}{\beta})\frac{\sqrt{x_2}\beta}{2\pi}]\mathrm{cosec}\frac{\sqrt{x_2}\beta}{2} - \frac{2\pi^2}{x_2\beta^2}\} + \frac{\Delta\beta^2}{4\pi^2}\{\frac{\pi^2}{\sqrt{x_3}\beta}\cos[(\pi - \frac{4\pi T_0}{\beta})\frac{\sqrt{x_3}\beta}{2\pi}]\mathrm{cosec}\frac{\sqrt{x_3}\beta}{2} - \frac{2\pi^2}{x_3\beta^2}\}\}. \tag{11}$$

The general semiclassical action with account for two promoting phonon modes is then reduced to the following:

$$S_B = 2\omega_0^2(a+b)a\tau_0 - \frac{2}{\beta}\omega_0^2(a+b)^2\tau_0^2 - \frac{4}{\beta}\omega_0^4(a+b)^2\{U_1 + U_2\}, \tag{12}$$

where $\tau_0^* = \tau_0\omega_0 = \mathrm{arcsinh}\left[\frac{b^*-1}{b^*+1}\sinh\beta^*\right]/2 + \beta^*;\ \beta^* = \omega_0\beta/4$.

Finally, the normalized expression for 1D semiclassical instanton action S_B on the basis of two local modes of the wide-band matrix takes the form Eq. (4) of our paper.

Let us turn to the calculation of the pre-exponential factor B with the account for the 2 promoting phonon modes.

$$B = \frac{2\omega_0^2(a+b)^2}{(2\pi\beta)^{\frac{1}{2}}}\sum_{n=-\infty}^{\infty}\frac{\sin^2\nu_n\tau_0}{\lambda_{0n}}/\left[\sum_{n=-\infty}^{\infty}\frac{\cos 2\nu_n\tau_0}{\lambda_{0n}}\right]^{\frac{1}{2}}, \tag{13}$$

where $\lambda_{0n} = \nu_n^2 + \omega_0^2 + \zeta_n$,

$$\sum_{n=-\infty}^{\infty}\frac{\sin^2\nu_n\tau_0 = \frac{1}{2}(1-\cos 2\nu_n\tau_0)}{\nu_n^2 + \omega_0^2 + \nu_n^2 C_2^2/\omega_2^2(\omega_2^2+\nu_n^2) + \nu_n^2 C_3^2/\omega_3^2(\omega_3^2+\nu_n^2)} = \frac{1}{2}\sum_{n=-\infty}^{\infty}\frac{(1-\cos 2\nu_n\tau_0)(\omega_2^2+\nu_n^2)(\omega_3^2+\nu_n^2)}{x^3 + Ax^2 + Bx + C}. \tag{14}$$

We expand the denominator of the last expression in Eq. (2) as follows:

$$\frac{1}{2}\sum_{n=-\infty}^{\infty}\frac{(\omega_2^2+\nu_n^2)(\omega_3^2+\nu_n^2)}{(\nu_n^2-x_1)(\nu_n^2-x_2)(\nu_n^2-x_3)} = \frac{D}{\nu_n^2-x_1} + \frac{E}{\nu_n^2-x_2} + \frac{F}{\nu_n^2-x_3}, \tag{15}$$

where

$$E=\frac{\omega_2^2+\omega_3^2+x_2+x_3+F(x_1-x_3)}{x_2-x_1},\quad D=-\frac{\omega_2^2+\omega_3^2+E(x_1+x_3)+F(x_1+x_2)}{x_2+x_3},$$

$$F=\frac{(\omega_2^2+\omega_3^2+x_2+x_3)[x_2x_3(x_1+x_3)-x_1x_3(x_2+x_3)]+(x_2-x_1)[(x_2+x_3)\omega_2^2\omega_3^2+x_2x_3(\omega_2^2+\omega_3^2)]}{(x_2-x_1)[x_1x_2(x_2+x_3)-x_2x_3(x_1+x_2)]-(x_1-x_3)[x_2x_3(x_1+x_3)-x_1x_3(x_2+x_3)]}. \tag{16}$$

So that

$$\frac{1}{2}\sum_{n=-\infty}^{\infty}\frac{D}{\nu_n^2-x_1}=\frac{D}{2}\sum_{n=-\infty}^{\infty}\frac{1}{\frac{4\pi^2n^2}{\beta^2}-x_1}=\frac{1}{2}\frac{D\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_1\beta^2}+2\sum_{n=-\infty}^{\infty}\frac{1}{n^2-x_1\beta^2/4\pi^2}\right]. \tag{17}$$

For $x_1>0$, we then have

$$\frac{1}{2}\frac{D\beta^2}{4\pi^2}\sum_{n=-\infty}^{\infty}\frac{1}{n^2-x_1\beta^2/4\pi^2}=\frac{1}{2}\frac{D\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_1\beta^2}+2\left\{-\frac{2\pi^2}{x_{10}\beta^2}-\frac{\pi^2}{\sqrt{x_1}\beta}\text{cotan}\frac{\sqrt{x_1}\beta}{2}\right\}\right] \tag{18}$$

and

$$\sum_{n=-\infty}^{\infty}\frac{\cos 2\nu_n\tau_0}{\lambda_{0n}}{}_2=-\frac{1}{2}\frac{D\beta^2}{4\pi^2}[-\frac{4\pi^2}{x_1\beta^2}+2\{-\frac{\pi^2}{\sqrt{x_1}\beta}\cos[(\pi-\frac{4\pi\tau_0}{\beta})\frac{\sqrt{x_1}\beta}{2\pi}]\text{cosec}\frac{\sqrt{x_1}\beta}{2}+\frac{2\pi^2}{x_1\beta^2}\}]. \tag{19}$$

In the result, the dimensionless pre-exponential factor B has the form of Eq. (12) of our paper.

So, after tedious calculations details, we get the following exact analytical expressions for the normalized action $\tilde{S}_B$ and pre-exponential factor $\tilde{B}$:

$$\tilde{S}_B=\frac{S_B}{\omega_0a^2}=2(b^*+1)\tau_0^*-\frac{1}{2\beta^*}(b^*+1)^2\tau_0^{*2}-\frac{(b^*+1)^2}{\beta^*}\left\{\frac{1}{2}\left[\beta_0\omega_0^2(\frac{\beta\omega_0}{4})^2\frac{2}{3}+4\frac{\gamma\omega_0^2(\frac{\beta\omega_0}{4})^2}{\pi^2}\left[-\frac{4\pi^2}{2x_1\beta^2}-\right.\right.\right.$$

$$\left.-\frac{\pi^2}{\sqrt{x_1}\beta}\text{cotan}(\frac{\sqrt{x_1}\beta}{2\pi})\right]+4\frac{\varphi\omega_0^2\beta^{*2}}{\pi^2}\left[-\frac{4\pi^2}{2x_2\beta^2}-\frac{\pi^2}{\sqrt{x_2}\beta}\text{cotan}(\frac{\sqrt{x_2}\beta}{2\pi})\right]+4\frac{\Delta\omega_0^2\beta^{*2}}{\pi^2}\left[-\frac{4\pi^2}{2x_3\beta^2}-\frac{\pi^2}{\sqrt{x_3}\beta}\text{cotan}(\frac{\sqrt{x_3}\beta}{2\pi})\right]\Bigg]-$$

$$-\frac{1}{2}\left[\beta_0\omega_0^2\left(\frac{\beta\omega_0}{4}\right)^2\frac{1}{3}\left(3\left(\frac{4\pi\tau_0\omega_0}{\beta\omega_0}\right)^2-\frac{6\pi^2\tau_0\omega_04}{\beta\omega_0}+2\pi^2\right)+\right.$$

$$+\frac{4\gamma\omega_0^2\left(\frac{\beta\omega_0}{4}\right)^2}{\pi^2}\left\{\frac{\omega_0\pi^24}{4\sqrt{x_1}\beta\omega_0}\cos\left[(\pi-\frac{4\pi\tau_0\omega_0}{\beta\omega_0})\frac{\sqrt{x_1}2\beta\omega_0}{\omega_0\pi4}\right]\text{cosec}\frac{2\sqrt{x_1}}{\omega_0}\frac{\beta_0\omega_0}{4}+\frac{\omega_0^2\pi^24}{8x_1\beta\omega_0}\right\}+$$

$$+\frac{4\phi\omega_0^2\beta^{*2}}{\pi^2}\left\{\frac{\omega_0\pi^24}{4\sqrt{x_2}\beta^*}\cos\left[(\pi-\frac{4\pi\tau_0^*\omega_0}{\beta^*})\frac{\sqrt{x_2}2\beta^*}{\omega_0\pi}\right]\text{cosec}\frac{2\sqrt{x_2}}{\omega_0}\beta^*+\frac{\omega_0^2\pi^2}{8x_{20}\beta^{*2}}\right\}+$$

$$\left.\left.+\frac{4\Delta\omega_0^2\beta^{*2}}{\pi^2}\left\{\frac{\omega_0\pi^24}{4\sqrt{x_3}\beta^*}\cos\left[(\pi-\frac{4\pi\tau_0^*\omega_0}{\beta^*})\frac{\sqrt{x_3}2\beta^*}{\omega_0\pi}\right]\text{cosec}\frac{2\sqrt{x_3}}{\omega_0}\beta^*+\frac{\omega_0^2\pi^2}{8x_3\beta^{*2}}\right\}\right]\right\} \tag{20}$$

and

$$\tilde{B}=\frac{B}{a^2\omega^{3/2}}=\frac{2\omega_0^2(b/a+1)^2}{\sqrt{2\pi\beta}}\frac{V_1}{\sqrt{V_2}}, \tag{21}$$

where

$$V_1=\sum_{n=-\infty}^{\infty}\frac{\sin^2\nu_n\tau_0}{\lambda_{0n}}=\frac{1}{2}\frac{D\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_1\beta^2}+2\left\{-\frac{2\pi^2}{x_{10}\beta^2}-\frac{\pi^2}{\sqrt{x_1}\beta}\text{cotan}\frac{\sqrt{x_1}\beta}{2}\right\}\right]+\frac{1}{2}\frac{E\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_2\beta^2}+\right.$$

$$\left.+2\left\{-\frac{2\pi^2}{x_{20}\beta^2}-\frac{\pi^2}{\sqrt{x_2}\beta}\text{cotan}\frac{\sqrt{x_2}\beta}{2}\right\}\right]+\frac{1}{2}\frac{F\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_3\beta^2}+2\left\{-\frac{2\pi^2}{x_{30}\beta^2}-\frac{\pi^2}{\sqrt{x_3}\beta}\text{cotan}\frac{\sqrt{x_3}\beta}{2}\right\}\right]-\frac{1}{2}V_2,$$

$$V_2=\sum_{n=-\infty}^{\infty}\frac{\cos^2\nu_n\tau_0}{\lambda_{0n}}=\frac{D\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_1\beta^2}+2\left\{-\frac{\pi^2}{\sqrt{x_1}\beta}\cos[(\pi-\frac{4\pi\tau_0}{\beta})\frac{\sqrt{x_1}\beta}{2\pi}]\mathrm{cosec}\frac{\sqrt{x_1}\beta}{2}+\frac{2\pi^2}{x_1\beta^2}\right\}\right]+$$
$$+\frac{E\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_2\beta^2}+2\left\{-\frac{\pi^2}{\sqrt{x_2}\beta}\cos[(\pi-\frac{4\pi\tau_0}{\beta})\frac{\sqrt{x_2}\beta}{2\pi}]\mathrm{cosec}\frac{\sqrt{x_2}\beta}{2}+\frac{2\pi^2}{x_2\beta^2}\right\}\right]+$$
$$+\frac{F\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_3\beta^2}+2\left\{-\frac{\pi^2}{\sqrt{x_3}\beta}\cos[(\pi-\frac{4\pi\tau_0}{\beta})\frac{\sqrt{x_3}\beta}{2\pi}]\mathrm{cosec}\frac{\sqrt{x_3}\beta}{2}+\frac{2\pi^2}{x_3\beta^2}\right\}\right]. \tag{22}$$

With the obtained analytical formulas (20)–(22), can directly derive dissipative quantum probability rate Γ given by Eq. (6) of our paper, which determines the current through the barrier, as a function of the external electric field E.

Non-oscillatory case. Solution for the case of non-oscillating character of the probability rate is

$$S_B=2(1+b)a\tau_0-\frac{1}{2\beta}\omega_0^2(1+b)^2\tau_0^{*2}-\frac{\omega_0^4(1+b)^2\{U_1-U_2\}}{\beta}, \tag{23}$$

where for $x_1,\, x_2,\, x_3<0$, $(x_{10},\, x_{20},\, x_{30}>0)$,

$$U_1=\frac{1}{2}\left\{\beta_0\frac{\beta^2}{24}+\frac{\gamma\beta^2}{4\pi^2}\left[\frac{1}{2\tilde{x}_{10}^2}+\frac{\pi}{2\tilde{x}_{10}}\mathrm{cotanh}\,(\pi\tilde{x}_{10})\right]+\right.$$
$$\left.+\frac{\phi\beta^2}{4\pi^2}\left[\frac{1}{2\tilde{x}_{20}^2}+\frac{\pi}{2\tilde{x}_{20}}\mathrm{cotanh}\,(\pi\tilde{x}_{20})\right]+\frac{\Delta\beta^2}{4\pi^2}\left[\frac{1}{2\tilde{x}_{30}^2}+\frac{\pi^2}{2\tilde{x}_{30}}\mathrm{cotanh}\,(\pi\tilde{x}_{30})\right]\right\}, \tag{24}$$

$$U_2=\frac{1}{2}\left\{\frac{\beta_0\beta^2}{48}\left(3\left(\frac{4\pi\tau_0}{\beta}\right)^2-\frac{24\pi^2\tau_0}{\beta}+2\pi^2\right)+\frac{\gamma\beta^2}{4\pi^2}\left\{\frac{\pi^2}{\sqrt{x_{10}}\beta}\mathrm{ch}\left[\left(\pi-\frac{4\pi\tau_0}{\beta}\right)\frac{\sqrt{x_{10}}\beta}{2\pi}\right]\mathrm{cosech}\frac{\sqrt{x_{10}}\beta}{2}-\frac{2\pi^2}{x_{10}\beta^2}\right\}+\right.$$
$$+\frac{\phi\beta^2}{4\pi^2}\left\{\frac{\pi^2}{\sqrt{x_{20}}\beta}\cosh\left[\left(\pi-\frac{4\pi\tau_0}{\beta}\right)\frac{\sqrt{x_{20}}\beta}{2\pi}\right]\mathrm{cosech}\frac{\sqrt{x_{20}}\beta}{2}-\frac{2\pi^2}{x_{20}\beta^2}\right\}+$$
$$\left.+\frac{\Delta\beta^2}{4\pi^2}\left\{\frac{\pi^2}{\sqrt{x_{30}}\beta}\cosh\left[\left(\pi-\frac{4\pi\tau_0}{\beta}\right)\frac{\sqrt{x_{30}}\beta}{2\pi}\right]\mathrm{cosech}\frac{\sqrt{x_{30}}\beta}{2}-\frac{2\pi^2}{x_{30}\beta^2}\right\}\right\}, \tag{25}$$

with B is given by Eq. (21), in which

$$V_1=\sum_{n=-\infty}^{\infty}\frac{\sin^2\nu_n\tau_0}{\lambda_{0n}}=\frac{1}{2}\frac{D\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_1\beta}+2\left\{\frac{2\pi^2}{x_{10}\beta^2}+\frac{\pi^2}{\sqrt{x_{10}}\beta}\mathrm{cotanh}\frac{\sqrt{x_{10}}\beta}{2}\right\}\right]+\frac{1}{2}\frac{E\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_2\beta^2}+\right.$$
$$\left.+2\left\{\frac{2\pi^2}{x_{20}\beta^2}+\frac{\pi^2}{\sqrt{x_{20}}\beta}\mathrm{cotanh}\frac{\sqrt{x_{20}}\beta}{2}\right\}\right]+\frac{1}{2}\frac{F\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_3\beta^2}+2\left\{\frac{2\pi^2}{x_{30}\beta^2}+\frac{\pi^2}{\sqrt{x_{30}}\beta}\mathrm{cotanh}\frac{\sqrt{x_{30}}\beta}{2}\right\}\right]-$$
$$-\frac{1}{2}\frac{D\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_1\beta^2}+2\left\{\frac{\pi^2}{\sqrt{x_{10}}\beta}\cosh\left[\left(1-\frac{4\tau_0}{\beta}\right)\frac{\sqrt{x_{10}}\beta}{2}\right]\mathrm{cosech}\frac{x_{10}\beta}{2}-\frac{2\pi^2}{x_{10}\beta^2}\right\}\right]-$$
$$-\frac{1}{2}\frac{E\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_2\beta^2}+2\left\{\frac{\pi^2}{\sqrt{x_{20}}\beta}\cosh\left[\left(1-\frac{\tau_0^*}{\beta^*}\right)\frac{\sqrt{x_{20}}\beta}{2}\right]\mathrm{cosech}\frac{\sqrt{x_{20}}\beta}{2}-\frac{2\pi^2}{x_{20}\beta^2}\right\}\right]-$$
$$-\frac{1}{2}\frac{F\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_3\beta^2}+2\left\{\frac{\pi^2}{\sqrt{x_{30}}\beta}\cosh\left[\left(1-\frac{4\tau_0}{\beta}\right)\frac{\sqrt{x_{30}}\beta}{2}\right]\mathrm{cosech}\frac{\sqrt{x_{30}}\beta}{2}-\frac{2\pi^2}{x_{30}\beta^2}\right\}\right], \tag{26}$$

$$V_2=\sum_{n=-\infty}^{\infty}\frac{\cos\nu_n\tau_0}{\lambda_{0n}}=\frac{D\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_1\beta}+\left\{\frac{\pi^2}{x_{10}\beta^2}+\frac{\pi^2}{\sqrt{x_{10}}\beta}\cosh\left[\left(1-\frac{4\tau_0}{\beta}\right)\frac{\sqrt{x_{10}}\beta}{2}\right]\mathrm{cosech}\frac{\sqrt{x_{10}}\beta}{2}-\frac{2\pi^2}{x_{10}\beta^2}\right\}\right]+$$
$$+\frac{1}{2}\frac{E\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_2\beta^2}+2\left\{\frac{2\pi^2}{x_{20}\beta^2}+\frac{\pi^2}{\sqrt{x_{20}}\beta}\mathrm{cotanh}\frac{\sqrt{x_{20}}\beta}{2}\right\}\right]+\frac{1}{2}\frac{F\beta^2}{4\pi^2}\left[-\frac{4\pi^2}{x_3\beta^2}+2\left\{\frac{2\pi^2}{x_{30}\beta^2}+\frac{\pi^2}{\sqrt{x_{30}}\beta}\mathrm{cotanh}\frac{\sqrt{x_{30}}\beta}{2}\right\}\right]. \tag{27}$$