Supplemental Material for
"Fermi points and the Nambu sum rule in the polar phase of ^{3}He"

1. "Gap" equation for the polar phase with the spin-orbit interaction taken into account. In the presence of spin-orbit interactions we consider the condensate of the form

$$A^{(0)}_{\alpha i} = (\beta V)^{1/2}\frac{\Delta}{2}\,\delta_{p0}\Big(\hat{d}^\alpha \hat{m}^i + \kappa^{\alpha i}\Big), \tag{1}$$

with $|\kappa^{\alpha i}| \ll 1$. The gap equation receives the form

$$\Omega^{i\alpha} \equiv \Big(\frac{1}{g} - \frac{1}{g_m}\Big)\kappa^{\alpha i}\Delta + \Big(\frac{1}{g} - \frac{1}{g_m} - \frac{2}{5}g_D\Big)\hat{m}^i\hat{d}^\alpha\Delta + \frac{3}{5}g_D\hat{m}^\alpha\hat{d}^i\Delta = -2\int\frac{d^3k d\omega}{(2\pi)^4}\mathrm{Tr}\gamma^5\gamma^\alpha\hat{k}^i G(\mathrm{i}\omega, k) \tag{2}$$

with

$$G(\epsilon, k) = \frac{1}{\sum_{\mu=1,2,3,5}\mathcal{P}_\mu(\epsilon,k)\gamma^\mu - \mathcal{M}(k)}\gamma^5 \tag{3}$$

and

$$\mathcal{P}^5 = \epsilon,\, \mathcal{P}^\alpha = \Delta\Big(\hat{d}^\alpha\hat{m}^i + \kappa^{\alpha i}\Big)\hat{k}^i,\, \mathcal{M} = v_{\mathrm{F}}(|k| - k_{\mathrm{F}}).$$

(It is taken into account that $(\hat{m}\hat{d}) = 0$.) We may rewrite this equation as follows

$$\Omega^{i\alpha} = -2\int\frac{d^3k d\omega}{(2\pi)^4}\frac{\mathrm{Tr}\Big(\mathcal{P}_\mu(\epsilon,k)\gamma^\mu + \mathcal{M}(k)\Big)\gamma^\alpha\hat{k}^i}{\omega^2 + \Delta_\theta^2 + \mathcal{M}^2(k)}, \tag{4}$$

where $\Delta_\theta = \Delta(\hat{m}\hat{k})$. Now we have

$$\begin{aligned}\Big(\frac{1}{g} - \frac{1}{g_m}\Big)\kappa^{\alpha i}\Delta &= \frac{2}{5}g_D\hat{m}^i\hat{d}^\alpha\Delta - \frac{3}{5}g_D\hat{m}^\alpha\hat{d}^i\Delta + \kappa^{\alpha i}\Delta\Big(\frac{1}{2}J^{(0)} - \frac{1}{2}J^{(1)}\Big) + (\kappa^{\alpha j}\hat{m}^j)\hat{m}^i\Delta\Big(\frac{3}{2}J^{(1)} - \frac{1}{2}J^{(0)}\Big) + \\ &+ (2\kappa^{\beta i}\hat{d}^\beta)\hat{d}^\alpha\Delta\Big(\frac{1}{2}\tilde{J}^{(0)} - \frac{1}{2}\tilde{J}^{(1)}\Big) + (2\kappa^{\beta j}\hat{d}^\beta\hat{m}^j)\hat{d}^\alpha\hat{m}^i\Delta\Big(\frac{3}{2}\tilde{J}^{(1)} - \frac{1}{2}\tilde{J}^{(0)}\Big),\end{aligned} \tag{5}$$

where

$$\begin{aligned}J^{(0)} &= \frac{1}{4\pi^2 v_{\mathrm{F}}^3}\int\frac{d\phi}{2\pi}d\cos\theta\int_{4\Delta_\theta^2}^{\Lambda_\theta^2}dt\,\frac{t - 4\Delta_\theta^2 + 4v_{\mathrm{F}}^2k_{\mathrm{F}}^2}{\sqrt{t - 4\Delta_\theta^2}\sqrt{t}},\\ J^{(1)} &= \frac{1}{4\pi^2 v_{\mathrm{F}}^3}\int\frac{d\phi}{2\pi}d\cos\theta\int_{4\Delta_\theta^2}^{\Lambda_\theta^2}dt\frac{t - 4\Delta_\theta^2 + 4v_{\mathrm{F}}^2k_{\mathrm{F}}^2}{\sqrt{t - 4\Delta_\theta^2}\sqrt{t}}(\hat{k}\hat{m})^2,\\ \tilde{J}^{(0)} &= \Delta^2\frac{\partial}{\partial\Delta^2}J^{(0)}, \quad \tilde{J}^{(1)} = \Delta^2\frac{\partial}{\partial\Delta^2}J^{(1)}.\end{aligned} \tag{6}$$

Here the energy cutoff Λ_θ and the momentum cutoff $\mathcal{K}$ are related by expression $\Lambda_\theta^2/4 = v_{\mathrm{F}}^2\mathcal{K}^2 + \Delta_\theta^2$ (integration is over momenta with $|k - k_{\mathrm{F}}| < \mathcal{K}$). We keep the terms linear in g_D and $\kappa^{\alpha i}$. Here

$$\begin{aligned}J^{(0)} &\approx \frac{4k_{\mathrm{F}}^2}{\pi^2 v_{\mathrm{F}}}\Big(\log\frac{2v_{\mathrm{F}}\mathcal{K}}{\Delta} + 1\Big),\\ J^{(1)} &= \frac{1}{g} - \frac{1}{g_m} \approx \frac{4k_{\mathrm{F}}^2}{3\pi^2 v_{\mathrm{F}}}\Big(\log\frac{2v_{\mathrm{F}}\mathcal{K}}{\Delta} + \frac{1}{3}\Big),\\ \tilde{J}^{(1)} &= -\frac{2k_{\mathrm{F}}^2}{3\pi^2 v_{\mathrm{F}}}\end{aligned} \tag{7}$$

and

$$\kappa^{\alpha i} = a\hat{d}^{\alpha}\hat{m}^{i} + b\hat{m}^{\alpha}\hat{d}^{i} \tag{8}$$

with $a = \frac{3v_{\rm F}\pi^2}{10}\frac{g_D}{k_{\rm F}^2}$ and $b = \frac{v_{\rm F}9\pi^2}{20}\frac{g_D}{k_{\rm F}^2}$.

2. Bosonic collective modes in the polar phase. Let us calculate the energy gaps of the bosonic collective modes. In our calculation for simplicity we neglect spin-orbit interaction. The quadratic part of the effective action for the fluctuations around the condensate has the form:

$$S^{(1)}_{\rm eff} = (\bar{u}, \bar{v})[1/g - \Omega - \Pi]\begin{pmatrix} u \\ v \end{pmatrix}, \tag{9}$$

where

$$\Omega^{\alpha i}_{\bar{\alpha}\bar{i}} = \frac{1}{g_m}\delta^{\alpha}_{\bar{\alpha}}\hat{m}^{i}\hat{m}^{\bar{i}},$$

while

$$u_{i\alpha}(p) = \frac{\delta A_{i\alpha}(p) + \delta\bar{A}_{i\alpha}(-p)}{2}$$

and

$$v_{i\alpha}(p) = \frac{\delta A_{i\alpha}(p) - \delta\bar{A}_{i\alpha}(-p)}{2i}.$$

Here

$$\Big[\Pi^{\bar{u}u}(E)\Big]^{\alpha i}_{\bar{\alpha}\bar{i}} = {\rm i}\int\frac{d^3k d\epsilon}{(2\pi)^4}{\rm Tr}\, G(\epsilon,k)\gamma^5\gamma^{\alpha}\hat{k}^{i}G(\epsilon - E,k)\gamma^5\gamma^{\bar{\alpha}}\hat{k}^{\bar{i}} \tag{10}$$

and

$$\Big[\Pi^{\bar{v}v}(E)\Big]^{\alpha i}_{\bar{\alpha}\bar{i}} = -{\rm i}\int\frac{d^3k d\epsilon}{(2\pi)^4}{\rm Tr}\, G(\epsilon,k)\gamma^{\alpha}\hat{k}^{i}G(\epsilon - E,k)\gamma^{\bar{\alpha}}\hat{k}^{\bar{i}}. \tag{11}$$

The polarization operator can be represented as

$$\Pi(E) = \frac{1}{\pi}\int_0^{\infty} dz\frac{\rho(z)}{z - E^2}, \tag{12}$$

where the spectral function may be calculated using the Cutkosky rule (see the Landau–Lifshitz course of theoretical physics, vol. 4, chapter 115)

$$\begin{aligned}
2\Big[\rho^{\bar{u}u}\Big]^{\alpha i}_{\bar{\alpha}\bar{i}} &= -4\pi^2\int_{\epsilon>0}\frac{d^3k d\epsilon}{(2\pi)^4}{\rm Tr}\,\Big(\mathcal{P}_{\mu}(\epsilon,k)\gamma^{\mu} + \mathcal{M}(k)\Big)\gamma^{\alpha}\hat{k}^{i}\Big(\mathcal{P}_{\mu}(\epsilon - E,k)\gamma^{\mu} + \mathcal{M}(k)\Big)\gamma^{\bar{\alpha}}\hat{k}^{\bar{i}}\times \\
&\times\delta(\mathcal{P}^2(\epsilon,k) - \mathcal{M}^2(k))\delta(\mathcal{P}^2(\epsilon - E,k) - \mathcal{M}^2(k)) = -\sum_{\pm}\int\frac{d\phi\Big(k_{\rm F} \pm \frac{\sqrt{t-4\Delta_\theta^2}}{2v_{\rm F}}\Big)^2 d\cos\theta}{2\pi 2\pi v_{\rm F}\sqrt{t - 4\Delta_\theta^2}\sqrt{t}}\times \\
&\times\Big((\frac{t}{2} - \Delta_\theta^2){\rm Tr}\gamma^{\alpha}\hat{k}^{i}_{\pm}\gamma^{\bar{\alpha}}\hat{k}^{\bar{i}}_{\pm} + \Delta_\theta^2{\rm Tr}(\hat{d}\gamma)\gamma^{\alpha}\hat{k}^{i}_{\pm}(\hat{d}\gamma)\gamma^{\bar{\alpha}}\hat{k}^{\bar{i}}_{\pm}\Big)\theta(t - 4\Delta_\theta^2)\theta(\Lambda_\theta^2 - t) = \\
&= \frac{1}{2\pi v_{\rm F}^3}\int\frac{d\phi}{2\pi}d\cos\theta\,\frac{t - 4\Delta_\theta^2 + 4v_{\rm F}^2k_{\rm F}^2}{\sqrt{t - 4\Delta_\theta^2}\sqrt{t}}\Big(t\delta^{\alpha\bar{\alpha}} - 4\Delta_\theta^2\hat{d}^{\alpha}\hat{d}^{\bar{\alpha}}\Big)\hat{k}^{i}_{+}\hat{k}^{\bar{i}}_{+}\theta(t - 4\Delta_\theta^2)\theta(\Lambda_\theta^2 - t),
\end{aligned} \tag{13}$$

where $\hat{k} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ while $E/2 = \sqrt{t}/2 = \epsilon_+ = \epsilon_-$; $k_\pm = k_{\rm F} \pm \frac{\sqrt{t-4\Delta_\theta}}{2v_{\rm F}}$, and $\Delta_\theta \equiv \Delta(\hat{m}\hat{k}_+) \equiv \Delta\cos\theta$. In the similar way

$$2\Big[\rho^{\bar{v}v}\Big]^{\alpha i}_{\bar{\alpha}\bar{i}} = \frac{1}{2\pi v_{\rm F}^3}\int\frac{d\,\phi\, d\cos\theta}{2\pi}\,\frac{t - 4\Delta_\theta^2 + 4v_{\rm F}^2k_{\rm F}^2}{\sqrt{t - 4\Delta_\theta^2}\sqrt{t}}\Big((t - 4\Delta_\theta^2)\delta^{\alpha\bar{\alpha}} + 4\Delta_\theta^2\hat{d}^{\alpha}\hat{d}^{\bar{\alpha}}\Big)\hat{k}^{i}_{+}\hat{k}^{\bar{i}}_{+}\theta(t - 4\Delta_\theta^2)\theta(\Lambda_\theta^2 - t). \tag{14}$$

3. Energy gaps and the Nambu sum rule. Let us come to the evaluation of the energy gaps.

$L=S=0$. We take components with $\alpha=2$, $i=3$. In the v-channel at $S=L=0$ the energy gap is equal to zero that leads to the condition

$$1/g-1/g_m=\int_{-1}^{1}\cos^2\theta d\cos\theta\int_{4\Delta_\theta^2}^{\Lambda_\theta^2}dt\frac{1}{4\pi^2v_{\mathrm F}^3}\frac{t-4\Delta_\theta^2+4v_{\mathrm F}^2k_{\mathrm F}^2}{\sqrt{t-4\Delta_\theta^2}\sqrt{t}}. \tag{15}$$

Recall that $\Delta_\theta=\Delta\cos\theta$ while the energy cutoff Λ_θ and the momentum cutoff $\mathcal{K}$ are related by expression $\Lambda_\theta^2/4=v_{\mathrm F}^2\mathcal{K}^2+\Delta_\theta^2$ (integration is over momenta with $|k-k_{\mathrm F}|<\mathcal{K}$). Actually, Eq. (15) is equivalent to the "gap" equation that relates the value of Δ with the coupling constants g, g_m and the momentum cutoff $\mathcal{K}$. In the similar way

$$1/g-1/g_m=\int_{-1}^{1}\cos^2\theta d\cos\theta\int_{4\Delta_\theta^2}^{\Lambda_\theta^2}dt\frac{1}{4\pi^2v_{\mathrm F}^3}\frac{t-4\Delta_\theta^2+4v_{\mathrm F}^2k_{\mathrm F}^2}{\sqrt{t-4\Delta_\theta^2}\sqrt{t}}\frac{t-4\Delta_\theta^2}{t-E_{u,L=0,S=0}^2}. \tag{16}$$

Let us subtract Eq. (15) from Eq. (16). Assuming that $v_{\mathrm F}k_{\mathrm F}\gg v_{\mathrm F}\mathcal{K}\gg\Delta$ we have:

$$0=\frac{2k_{\mathrm F}^2}{\pi^2v_{\mathrm F}}\int_{-1}^{1}\cos^2\theta d\cos\theta\int_{1}^{\infty}dz\frac{1}{\sqrt{z^2-1}}\frac{E_{u,L=0,S=0}^2/(4\Delta_\theta^2)-1}{z^2-E_{u,L=0,S=0}^2/(4\Delta_\theta^2)}. \tag{17}$$

The integrals in this equation may be taken and the result is expressed through the hypergeometric functions:

$$0=\frac{4k_{\mathrm F}^2}{\pi^2v_{\mathrm F}}\Big[\frac{1}{4}w^4\sqrt{\pi}\Big(\frac{3}{8w}\pi^{3/2}-\frac{32}{15\sqrt{\pi}}F_{3/2,7/2}^{1/2,1,3}(-w^2)\Big)-\frac{1}{4}w^4\sqrt{\pi}\Big(\frac{1}{2w}\pi^{3/2}-\frac{8}{3\sqrt{\pi}}F_{3/2,5/2}^{1/2,1,2}(-w^2)\Big)+\frac{1}{3}w^2+\frac{1}{3}\Big], \tag{18}$$

where

$$w=\frac{-\mathrm{i}E_{u,L=0,S=0}}{2\Delta}.$$

Technically we calculate the value of the integral in Eq. (17) at real values of w. Next, the obtained result is to be continued analytically to the whole complex plane. It is done in the way utilised inside the MAPLE package.

Numerical solution of this equation gives

$$E_{u,S=0,L=0}=\sqrt{12/5}\Big(1.007853779-0.3828669418\,i\Big)\Delta. \tag{19}$$

This solution is illustrated by Fig. 1, where the absolute value of the right hand side of Eq. (18) in the units of $\frac{4k_{\mathrm F}^2}{\pi^2v_{\mathrm F}}$ is represented as a function of $w=A+\mathrm{i}B$. One can see, that there is the solution in the physical part of the complex plane (at $\mathrm{Re}\,\omega<0$, $\mathrm{Im}\,\omega<0$). It corresponds to the energy gap of the given collective mode.

$L=0$, $S=1$. We take components with $\alpha=1,3$, $i=3$.

In the u-channel at $L=0$, $S=1$ the energy gap is equal to zero that leads to the condition, which coincides with Eq. (15). In the similar way equation for the v channel gives

$$E_{u,S=1,L=0}=0,\quad E_{v,S=1,L=0}=E_{u,S=0,L=0} \tag{20}$$

$L=1$, $S=0$. We take components with $\alpha=2$, $i=1,2$. In the u channel

$$\frac{1}{g}=\int_{-1}^{1}\frac{1-\cos^2\theta}{2}d\cos\theta\int_{4\Delta_\theta^2}^{\Lambda_\theta^2}dt\frac{1}{4\pi^2v_{\mathrm F}^3}\frac{t-4\Delta_\theta^2+4v_{\mathrm F}^2k_{\mathrm F}^2}{\sqrt{t-4\Delta_\theta^2}\sqrt{t}}\frac{t-4\Delta_\theta^2}{t-E_{u,L=1,S=0}^2}. \tag{21}$$

At $E_{u,L=1,S=0}=0$ we may rewrite this equation in the form with the integration over k instead of integration over t:

$$\frac{1}{g}=8\pi\int\frac{d^3k}{(2\pi)^4}\frac{\sin^2\theta\mathcal{M}^2(k)}{2(\Delta^2\cos^2\theta+\mathcal{M}^2(k))^{3/2}}. \tag{22}$$

One can check that after the integration over θ the right hand sides of the two expressions Eq. (4) and Eq. (22) coincide. Therefore, in the absence of the extra interaction that stabilizes direction of $\hat{m}$ in this channel the Goldstone boson appears as it should.

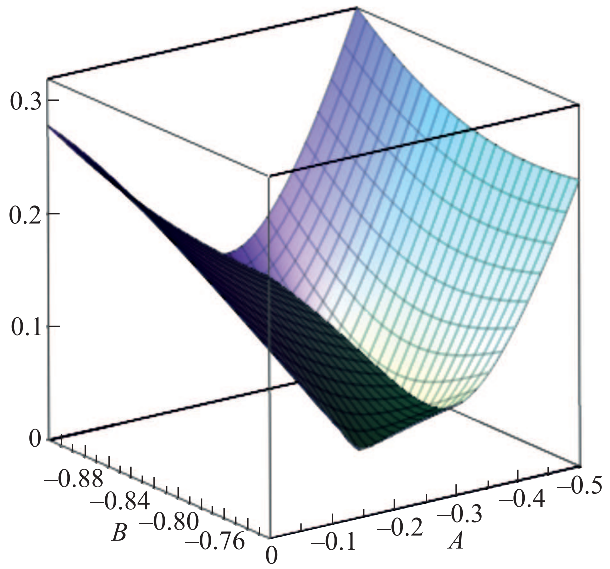

Figure 1: The absolute value of the right hand side of Eq. (18) in the units of $\frac{4k_{\rm F}^2}{\pi^2 v_{\rm F}}$ is represented as a function of $w = A + {\rm i}B$

In the presence of this extra interaction we have the following equation for the determination of $E_{u,L=1,S=0}$:

$$\frac{1}{g_m} = \int_{-1}^{1} \frac{1-\cos^2\theta}{2} d\cos\theta \int_{4\Delta_\theta^2}^{\Lambda_\theta^2} dt \frac{1}{4\pi^2 v_{\rm F}^3} \frac{t - 4\Delta_\theta^2 + 4v_{\rm F}^2 k_{\rm F}^2}{\sqrt{t-4\Delta_\theta^2}\sqrt{t}} \frac{\left(t-4\Delta_\theta^2\right) E_{u,L=1,S=0}^2}{t(t-E_{u,L=1,S=0}^2)}. \tag{23}$$

In the v channel we have

$$\frac{1}{g} = \int_{-1}^{1} \frac{1-\cos^2\theta}{2} d\cos\theta \int_{4\Delta_\theta^2}^{\Lambda_\theta^2} dt \frac{1}{4\pi^2 v_{\rm F}^3} \frac{t - 4\Delta_\theta^2 + 4v_{\rm F}^2 k_{\rm F}^2}{\sqrt{t-4\Delta_\theta^2}\sqrt{t}} \frac{t}{t-E_{v,L=1,S=0}^2}. \tag{24}$$

Subtracting the gap equation we may represent this expression as follows

$$\frac{1}{g_m} = \int_{-1}^{1} \frac{1-\cos^2\theta}{2} d\cos\theta \int_{4\Delta_\theta^2}^{\Lambda_\theta^2} dt \frac{1}{4\pi^2 v_{\rm F}^3} \frac{t - 4\Delta_\theta^2 + 4v_{\rm F}^2 k_{\rm F}^2}{\sqrt{t-4\Delta_\theta^2}\sqrt{t}} \frac{\left(E_{v,L=1,S=0}^2(t-4\Delta_\theta^2) + 4\Delta_\theta^2 t\right)}{t(t-E_{v,L=1,S=0}^2)}. \tag{25}$$

The value of $1/g_m$ should be sufficiently large in order to make vacuum stable. The critical value $g_m^{(c)}$ is determined by equation:

$$\frac{1}{g_m^{(c)}} = \int_{-1}^{1} \frac{1-\cos^2\theta}{2} d\cos\theta \int_{4\Delta_\theta^2}^{\Lambda_\theta^2} dt \frac{1}{4\pi^2 v_{\rm F}^3} \frac{t - 4\Delta_\theta^2 + 4v_{\rm F}^2 k_{\rm F}^2}{\sqrt{t-4\Delta_\theta^2}\sqrt{t}} \frac{4\Delta_\theta^2}{t} = \frac{2k_{\rm F}^2}{3\pi^2 v_{\rm F}}. \tag{26}$$

At this critical value of g_m the energy gap $E_{v,L=1,S=0}$ is close to zero. We get

$$-1/g_m + 1/g_m^{(c)} = \frac{2k_{\rm F}^2}{\pi^2 v_{\rm F}} \int_{-1}^{1} \frac{1-x^2}{2} dx \int_{1}^{\infty} dz \, \frac{1}{\sqrt{z^2-1}} \frac{w^2 + x^2}{x^2 z^2 + w^2}, \quad w = -\frac{iE_{u,L=1,S=0}}{2\Delta} \tag{27}$$

and

$$-1/g_m + 1/g_m^{(c)} = \frac{2k_{\rm F}^2}{\pi^2 v_{\rm F}} \int_{-1}^{1} \frac{1-x^2}{2} dx \int_{1}^{\infty} dz \, \frac{1}{\sqrt{z^2-1}} \frac{w^2}{x^2 z^2 + w^2}, \quad w = -\frac{iE_{v,L=1,S=0}}{2\Delta}. \tag{28}$$

The integration gives correspondingly

$$0 = 1/g_m - 1/g_m^{(c)} + \frac{4k_{\rm F}^2}{\pi^2 v_{\rm F}}\Big[\frac{1}{16}w^4\sqrt{\pi}\Big(\frac{1}{4w}\pi^{3/2} - \frac{16}{15\sqrt{\pi}}F^{1/2,1,2}_{3/2,7/2}(-w^2)\Big) + \\ + \frac{1}{16}w^4\sqrt{\pi}\Big(\frac{1}{w}\pi^{3/2} - \frac{8}{3\sqrt{\pi}}F^{1/2,1,1}_{3/2,5/2}(-w^2)\Big) - \frac{1}{6}w^2 + \frac{1}{3}\Big] \tag{29}$$

for the u-mode and

$$0 = 1/g_m - 1/g_m^{(c)} + \frac{4k_{\rm F}^2}{\pi^2 v_{\rm F}}\Big[\frac{1}{8}w^4\sqrt{\pi}\Big(\frac{1}{2w}\pi^{3/2} - \frac{8}{3\sqrt{\pi}}F^{1/2,1,2}_{3/2,5/2}(-w^2)\Big) + \\ + \frac{1}{8}w^4\sqrt{\pi}\Big(\frac{1}{w}\pi^{3/2} - \frac{4}{\sqrt{\pi}}F^{1/2,1,1}_{3/2,3/2}(-w^2)\Big) - \frac{1}{2}w^2\Big] \tag{30}$$

for the v-mode.

It appears, that for $1/g_m^{(c)} > 0 > 1/g_m$ the first equation has the solution for real value of w and imaginary value of $E_{u,L=1,S=0}$. For $0 = 1/g_m$ the solution with $E_{u,L=1,S=0} = 0$ appears, while for $0 < 1/g_m$ there are no solutions of this equation in the physical region of ω. (For $\mathrm{Im}\,\omega = 0$ the physical region is $\mathrm{Re}\,\omega \geq 0$.)

The second equation for $1/g_m < 1/g_m^{(c)}$ has the solution with real w and pure imaginary $E_{v,L=1,S=0}$, as it was pointed out above. For $1/g_m^{(c)} = 1/g_m$ the solution with $E_{v,L=1,S=0} = 0$ appears, while for $1/g_m^{(c)} < 1/g_m$ there is the solution with real negative w. It does not represent any solution of the original equation given by the integral and therefore belongs to the unphysical region of w.

This situation is illustrated by Fig. 2, where the absolute value of the right hand side of Eq. (30) in the units of $\frac{4k_{\rm F}^2}{\pi^2 v_{\rm F}}$ is

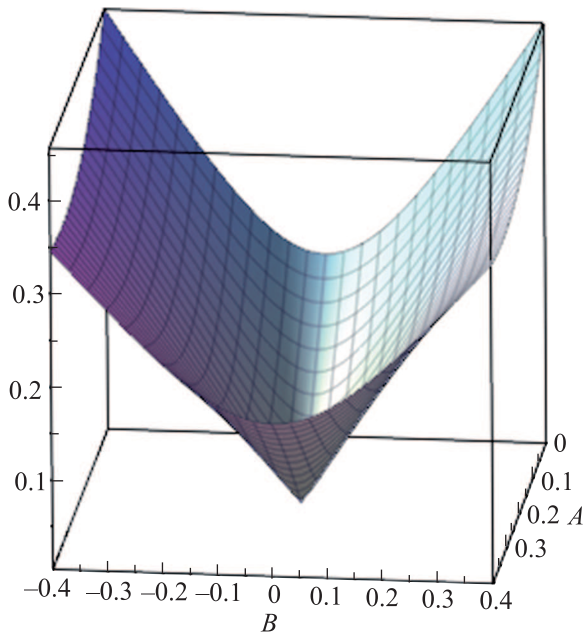

Figure 2: The absolute value of the right hand side of Eq. (30) in the units of $\frac{4k_{\rm F}^2}{\pi^2 v_{\rm F}}$ is represented as a function of $w = A + \mathrm{i}B$ for $1/g - 1/g_m = -0.2\frac{4k_{\rm F}^2}{\pi^2 v_{\rm F}}$

represented as a function of $w = A + \mathrm{i}B$ for $1/g_m - 1/g_m^{(c)} = -0.2\frac{4k_{\rm F}^2}{\pi^2 v_{\rm F}}$. One can see, that there is the solution at $\mathrm{Im}\,\omega = 0$, $\mathrm{Re}\,\omega > 0$. It corresponds to the pure imaginary energy gap, and indicates the instability of vacuum. When the value of

$1/g_m - 1/g_m^{(c)}$ is increased, the solution approaches zero. At $1/g_m - 1/g_m^{(c)} > 0$ the solution of Eq. (30) exists at real negative values of ω that are not physical because they do not correspond to any solutions of Eq. (28).

$L = 1, S = 1$. We take components with $\alpha = 1, 3$; $i = 1, 2$. It appears, that here the equations for the determinations of the gaps are the same as for $L = 1, S - 0$ with the modes u and v exchanged. We come to

$$E_{u,S=1,L=1} = E_{v,S=0,L=1}, \quad E_{v,S=1,L=1} = E_{u,S=0,L=1}. \tag{31}$$

One can see, that in the channels with $L = 0$, where the gaps of the order of Δ appear, these gaps satisfy the Nambu sum rule

$$E_u^2 + E_v^2 = 4\langle\Delta_\theta^2\rangle = \frac{12}{5}\Delta^2.$$

We come to the conclusion, that vacuum becomes stable for $g_m < g_m^{(c)}$, but the Higgs modes in the channels with $L = 1$ do not exist.