Supplemental Material to the article
"Limit velocity and dispersion law for domain walls in ferrimagnets in the vicinity of the spin compensation point"

1. Graphical analysis of the Eq. (11). On the Fig. S1, the dependency of DW momentum (upper panel) and energy (lower panel) are plotted according the Eq. (11) of the Letter. Here the values of parameters are the same as on the figures in the main part of the Letter: $\rho = 0.5$, and values of the uncompensation parameter $\bar{\nu}$ are $\bar{\nu} = 0.2 < \nu_c$ (red curves), $\bar{\nu} = \bar{\nu}_c$ (blue curves), and $\bar{\nu} = 1 > \bar{\nu}_c$ (dark green curves). Vertical green dash lines at $\varphi = \pm\pi/2$ border one period of the dependence $E(P)$. The values of $P_{\max}$ and $E_{\max}$ correspond to $\varphi = \varphi_{\max}$, where $\cos 2\varphi_{\max} = -\bar{\nu}^2/\rho = -(\nu/\nu_c)^2$. In the "antiferromagnetic" limit, $\nu \to 0$, the value of $\sin 2\varphi_{\max} \to 1$, both $P_{\max}$ and $E_{\max}$ diverge, the function $E(P)$ is not bounded from above and the ending point of the spectrum $E(P)$ is not present.

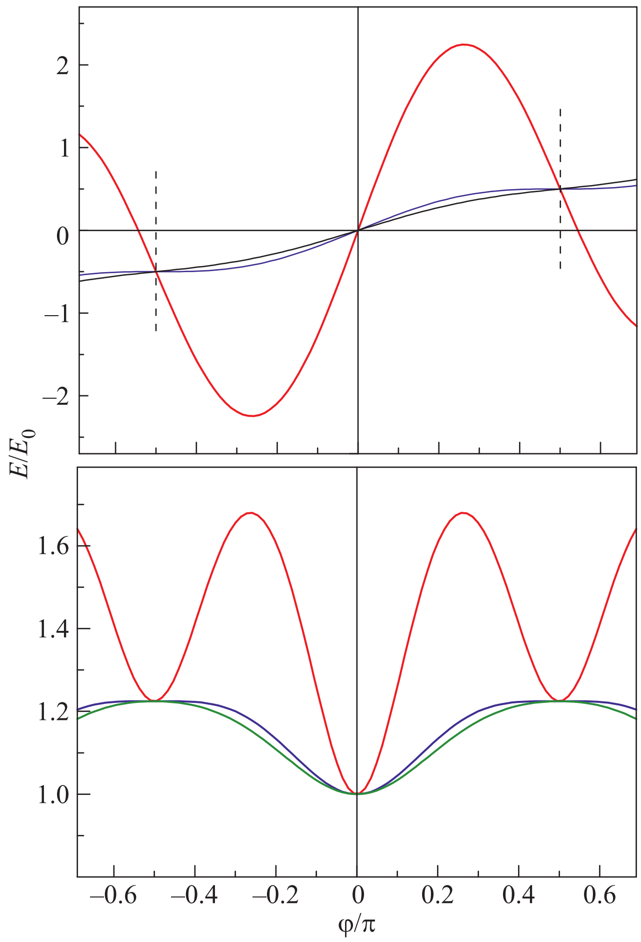

Fig. S1. The dependencies $P(\varphi)$ (upper panel, in units of the period P_0) and $E(\varphi)$ (lower panel, in units of E_0) for different values of ν (described in the text)

2. Analysis of the small perturbations on the ground of the domain wall. It is convenient to rewrite the equations (12) from the Letter through first order differential operators, $L^+ = -d/dx + \tanh x$ and $L^- = d/dx + \tanh x$, $\hat{L^-}\psi_0 = 0$, $\mathcal{H} = \hat{L^+}\hat{L^-}$, as the following

$$(\mathcal{H} - \omega^2)f - vG\hat{L^+}g = ig\omega G, \tag{1}$$
$$(\mathcal{H} - \omega^2 + B\cos 2\varphi)g - vG\hat{L^-}f = -if\omega G.$$

The functions f, g can be present in the following form, $f = f_0\psi_0 + \alpha$, $g = g_0\psi_0 + \beta$, with the condition $< \psi_0, \alpha >= 0$ and $< \psi_0, \beta >= 0$. For definiteness, the value $f_0 = 1$ can be chosen. Here the notation for "scalar product" of the functions,

$$< f_1, f_2 >= \int f_1^* f_2 dx, \tag{2}$$

is used. Equations for α and β take the form

$$(\mathcal{H} - \omega^2)\alpha - vG\hat{L^+}\beta - ig\omega G\beta = [\omega^2 f_0 + iG\omega g_0 + vGg_0\hat{L^+}]\psi_0, \tag{3}$$

$$(\mathcal{H} - \omega^2 + B\cos 2\varphi)\beta - vG\hat{L^-}\alpha + i\omega G\alpha = \psi_0[(\omega^2 - B)g_0 - iG\omega f_0]. \tag{4}$$

Applying the right product by $< \psi_0, ... >$ to the Eq. (3), we obtain the following exact connection between the amplitudes f_0 and g_0, $f_0\omega^2 + i\omega g_0 = 0$. Here we use the condition $< \psi_0, \hat{L^+}\alpha >=< \hat{L^-}\psi_0, \alpha >= 0$. Thus, the ratio g_0/f_0 is proportional to the frequency ω and it is small if $\omega \to 0$. Then doing the same operation with the Eq. (4), we can present the frequency through the matrix element with unknown yet function α

$$g_0(\omega^2 - B - G^2) + vG < \psi_0, \hat{L^-}\alpha >= 0. \tag{5}$$

Thus our goal is to calculate this matrix element. The equations for α, β with use of the above-obtained connection between f_0 and g_0 can be present as a set of non-uniform equations,

$$(\mathcal{H} - \omega^2)\alpha - vG\hat{L^+}\beta - ig\omega G\beta = vGg_0\hat{L^+}\psi_0, \tag{6}$$

$$\begin{aligned}(\mathcal{H} - \omega^2 + B\cos 2\varphi)\beta - vG\hat{L^-}\alpha + i\omega G\alpha + \\ + vG < \psi_0, \hat{L^-}\alpha > \psi_0 = 0.\end{aligned} \tag{7}$$

It is important to note that the only smaller (proportional to ω) amplitude g_0 is present in the RHS, and the functions α, β are proportional to ω. Looking for the instability point, where the value of ω^2 change its sign, consider the case of small $\omega \to 0$. In the linear approximation on ω the terms with $i\omega\alpha$ and $i\omega\beta$ can be neglected in the equations (6) and (7), and the equation (6) can be present as

$$\hat{L^+}[\hat{L^-}\alpha - vG\beta - vGg_0\psi_0] = 0. \tag{8}$$

The solution of the equation $\hat{L^+}F(x) = 0$ is proportional to $\cosh x$; thus, the expression in the square brackets should equals to zero. Substituting $\hat{L^-}\alpha = vG\beta + vGg_0\psi_0$ to the equation (7), we obtain the closed equation for the variable β only

$$(\mathcal{H} + B\cos 2\varphi - v^2G^2)\beta = \psi_0(vG- < \psi_0, \hat{L^-}\alpha >). \tag{9}$$

Taken the right product of (7) by ψ_0, and using the orthogonality condition $< \psi_0, \beta >= 0$, we obtain the exact formula for the matrix element in Eq. (5), $< \psi_0, \hat{L^-}\alpha >= vG$. Substituting this matrix element to the equation (5), the frequency can be present as the following

$$\omega^2 = (1 - v^2)G^2 + B\cos 2\varphi. \tag{10}$$

Note that the calculations done here are not the perturbation theory on the term vG; the only smallness of ω has been used. The accounting for the nonzero value of α, which leads to the appearance of the multiplier $(1 - v^2)$ before G^2, is significant for the analysis of non-small values of velocity $v \leq c$. Using the concrete formulae for B and G [Eq. (13) in the Letter], we arrive to the desired equation for the frequency, Eq. (14) from the Letter.

3. Forced motion of the DW with accounting for dissipation. The forced dynamics of the DW under an action of the driving force F of any origin and friction force F_{diss} can be described by Newton equation $dP/dt = F_{\text{diss}} + F$. If the driving force is caused by magnetic field, parallel to the easy axis ($z-$ axis), the force $F = 2M_sH_z$, it is finite at the spin compensation point (SCP). The friction force can be found through the dissipative function of the magnet, Q, as the following, $F_{\text{diss}}(v) = -2Q/v$. For the standard Gilbert dissipative function, which is proportional to $\alpha_G(\partial \mathbf{l}/\partial t)^2$, where α_G is phenomenological constant (Gilbert constant), and for the exact solution found in the main part of the Letter, the value of Q is proportional to the DW energy,

$$Q = v^2 \frac{4\alpha_G \hbar s_0}{l_0} \sqrt{\frac{1 + \rho \sin^2\varphi}{1 - v^2/c^2}}. \tag{11}$$

In the limit case of weak force, the linear dependence of the DW velocity on the driving force can be found, $v = \mu F$, where μ is the DW mobility, $\mu = l_0/(4\alpha_G \hbar s_0)$. Note that the value of mobility corresponding to the driving field F is independent on ν, see Fig. S2. Thus even for the case of magnetic field as a driving force, for $\nu \ll 1$ the mobility is weakly dependent on the uncompensation parameter ν. This feature agrees well with the data of numerical simulations of low-field regime of DW dynamics, see the inset on the Fig. 4a of the [18] in the main part of the Letter.

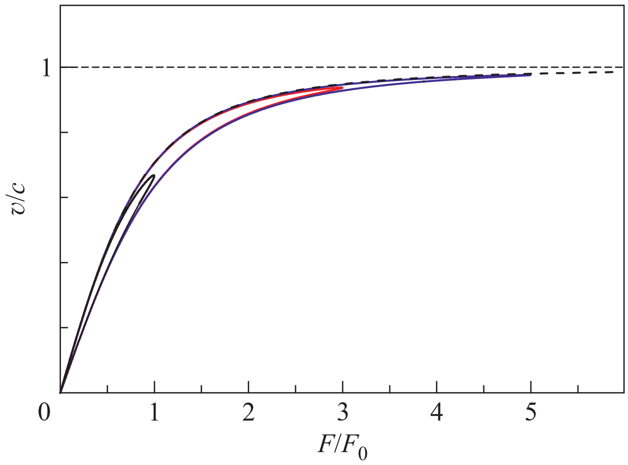

Fig. S2. Schematic dependence of the DW velocity on the driving force for $\rho = 0.5$ and different values of the parameter $F_{\max}$. The dashed line corresponds to the "antiferromagnetic" limit $\nu = 0$ for the Bloch wall. Here parameter F_0 is independent on ν and present through the value of the maximal force $F_{\max}$, $F_0 = F_{\max}(\nu/\nu_c)$. Thus the larger values of $F_{\max}/F_0$ corresponds to smaller values of ν

If the force is growing, the dependence $v(F)$ becomes more complicated. In the antiferromagnetic limit, $\nu = 0$, the saturating dependence of the form of $v(F) = \mu F c/\sqrt{(\mu F)^2 + c^2}$ appears, see Fig. S2. Such dependence is caused by Lorentz-invariance of the sigma-model for AFM, it has been observed of the DW dynamics in orthoferrites, see Refs. [14, 15] in the main text. At any finite ν, this Lorentz-invariance is broken, limit velocity $v_c < c$ and the value of the dissipative force (11) is limited from the up, $F_{\text{diss}}(v) \le F_{\max}$ at $v \le v_c$, where $F_{\max} = \alpha_G \rho K/2\nu$. Here, again, the non-analytical dependence on the parameters ρ and ν appears. The velocity of this steady-state motion can be written as

$$v = \frac{\mu F c}{\sqrt{(\mu F)^2 + (c^2/2)(2 + \rho \mp \rho\sqrt{1 - F^2/F_{\max}^2})}}, \tag{12}$$

where signs minus and plus correspond to Bloch and Neel walls. This dependence is plotted on the Fig. S2 for different values of ν. The discussion of non-stationary motion, which appears at $F > F_{\max}$, going beyond the scope of this Letter.