Supplemental Material to the article
"Two-impurity scattering in quasi-one-dimensional systems"

Evaluation of nonresonant scattering contributions $\rho_{\rm typ}$ and $\Delta\rho_{\rm log}$. As we have seen, for evaluation of $\rho_{\rm twin}$ it is enough to expand cot (for $\varepsilon > 0$) or coth (for $\varepsilon < 0$) to the lowest order in its argument, but, on the other hand, the $1/4\lambda$ shift of this argument, as well as the unity term in the denominator, are essential. For finding the rest of nonresonant contributions (i.e., for $\rho_{\rm typ}$ and $\Delta\rho_{\rm log}$), the expansion of coth is not sufficient, so one should keep coth as it is. However, there are some alternative simplifications that allow for the evaluation of these terms. Namely, one can neglect unity in the denominator of (22) and the shift $1/4\lambda$ of the arguments of cot. The reminiscence of the latter shift, however, appears as an ultraviolet cutoff of certain logarithmically divergent integrals. We have to note that thus proposed simplified method of calculation does not allow for reliable determination of the numerical factors under logarithms, more sophisticated approaches are needed for that end. Anyway, the contributions of these numerical factors are relatively small.

Having in mind that the above approximation does not work for $\rho_{\rm twin}$, we should first eliminate the corresponding part from the integrand. Then for $\Delta\rho \equiv \rho - \rho_{\rm twin}$ we get

$$\frac{\Delta\rho}{\rho_0} \approx \int_0^\infty e^{-n(L+L')} n^2 dL dL' [(q(L)+q(L')]^2 \approx 2\int_0^\infty e^{-nL}\left[q^2(L) - (4\lambda L)^{-2}\right] n dL + 2\left[\int_{1/2\lambda}^\infty e^{-nL} q(L) n dL\right]^2 \approx$$
$$\approx \frac{1}{8u_0^2}\left[2uF_2(u) + [F_1(u) + \ln u_0]^2\right], \qquad F_m(u) = \int_0^\infty e^{-t/2u}\left(\coth^m t - \frac{1}{t^m}\right) dt, \qquad u = \frac{\pi\sqrt{-\varepsilon}}{n}.$$

The functions $F_{1,2}(u)$ can be expressed in terms of digamma function ψ:

$$F_1(u) = \ln(1/4u) - \psi(1/4u) - 2u, \qquad F_2(u) = 1 + 2u + (1/2u)\ln 4u + (1/2u)\psi(1/4u).$$

Their asymptotics can be easily found

$$F_2(u) \approx \begin{cases} 2u - 1 + (1/2u)\ln u, & u \gg 1, \\ 4u/3, & u \ll 1, \end{cases} \qquad F_1(u) \approx \begin{cases} 2u - \ln u, & u \gg 1, \\ (4/3)u^2, & u \ll 1. \end{cases}$$

We should stress that $u \ll 1$ asymptotics are valid for both signs of ε, while the obtained $u \gg 1$ asymptotics are applicable only for $\varepsilon < 0$ case. The case $\varepsilon > 0$ and $u \gg 1$ is not relevant, since the nonresonant scattering is marginal in this energy domain.

To single out logarithmic contributions in $F_1(u)$, $uF_2(u)$ (which are related to $\Delta\rho_{\rm log}$) it is convenient to introduce the following modified functions:

$$\tilde{F}_1(u) = \frac{1}{4}\ln(1+u^4) + \int_0^\infty e^{-t/2u}\left(\coth t - \frac{1}{t}\right) dt \approx \begin{cases} 2u, & u \gg 1, \\ (4/3)u^2, & u \ll 1, \end{cases} \tag{1}$$

$$\tilde{F}_2(u) = -\frac{1}{8(1+u^4)^{1/4}}\ln(1+u^4) + \int_0^\infty e^{-t/2u}\left(\coth^2 t - \frac{1}{t^2}\right) dt \approx \begin{cases} 2u, & u \gg 1, \\ 4u/3, & u \ll 1, \end{cases} \tag{2}$$

which, in contrast with $F_1(u)$, $uF_2(u)$, are logarithm-free. Now the logarithmic contributions that belong to $\Delta\rho_{\rm log}$ can be easily separated from the logarithm-free ones, belonging to $\rho_{\rm typ}$:

$$\Delta\rho = \rho_{\rm typ} + \Delta\rho_{\rm log}, \quad \frac{\Delta\rho_{\rm typ}}{\rho_0} \approx \frac{1}{8u_0^2}\left[2u\tilde{F}_2(u) + \tilde{F}_1^2(u)\right] \approx \frac{u^2}{u_0^2}\begin{cases} 1, & u \gg 1, \\ 8/3, & u \ll 1, \end{cases} \tag{3}$$

$$\frac{\Delta\rho_{\rm log}}{\rho_0} \approx \frac{1}{8u_0^2}\left[\frac{u}{4(1+u^4)^{1/4}}\ln(1+u^4) + 2\tilde{F}_1(u)\ln\left[\frac{u_0}{(1+u^4)^{1/4}}\right] + \ln^2\left[\frac{u_0}{(1+u^4)^{1/4}}\right]\right] \approx$$
$$\approx \frac{1}{u_0^2}\begin{cases} (u/2)\ln(u_0/u) + \ln^2(u_0/u), & u \gg 1, \\ (u^2/3)\ln u_0 + (1/8)\ln^2 u_0, & u \ll 1. \end{cases} \tag{4}$$

Note that the choice (1), (2) is not unique: in principle, one could have introduced the "counter-logarithm" terms in a form $\frac{1}{\alpha}\ln(1+u^{\alpha})$ for $\tilde{F}_1(u)$ and $\frac{1}{2\alpha(1+u^{\alpha})^{1/\alpha}}\ln(1+u^{\alpha})$ for $\tilde{F}_2(u)$ with any $\alpha > 1$. We have chosen rather high $\alpha = 4$ in order not to affect the $u \ll 1$ asymptotics.