Supplementary Material to the article

"Anomalous radiative heating of a metal particle moving in close proximity to a metal plate"

A. General relativistic formula for the rate of particle heating. In the problem of relativistic fluctuation-electromagnetic interaction of a small dipole particle moving with a constant velocity V parallel to the surface of a thick plate with frequency-dependent dielectric permittivity ε and magnetic permittivity μ, one should discriminate the quantities relating to different inertial reference frames: the particle frame of rest, and that of the plate (laboratory system). The former one is co-moving with velocity V in the x-direction of the Cartesian coordinate system, associated with the plate. Then the rate of particle heating dQ'/dt' in its frame of rest (i.e. local rate of heating) is given by [13]

$$\frac{dQ'}{dt'} = \frac{\hbar\gamma^3}{2\pi^2}\int_0^\infty d\omega \int d^2k\omega^+ \left[\alpha_e''(\gamma\omega^+)\mathrm{Im}\left(\frac{e^{-2q_0z_0}}{q_0}R_e(\omega,\mathbf{k})\right) + (e\to m)\right]\left(\coth\frac{\hbar\omega}{2T_2} - \coth\frac{\hbar\gamma\omega^+}{2T_1}\right), \quad \text{(S1)}$$

where $\alpha''_{e,m}(\omega)$ are the imaginary parts of the particle electrical and magnetic polarizabilities, $\gamma = (1 - V^2/c^2)^{-1/2} = (1-\beta^2)^{-1/2}$ is the Lorentz factor, $\omega^+ = \omega + k_xV$, $\mathbf{k} = (k_x, k_y)$, $q_0 = (k^2 - \omega^2/c^2)^{1/2}$, $q = (k^2 - \varepsilon\mu\omega^2/c^2)^{1/2}$, $R_e(\omega,\mathbf{k}) = A_1\Delta_e(\omega) + A_2\Delta_m(\omega)$, $R_m(\omega,\mathbf{k}) = A_1\Delta_m(\omega) + A_2\Delta_e(\omega)$,

$$A_1 = 2(k^2 - k_x^2\beta^2)(1-\omega^2/k^2c^2) + \frac{(\omega^+)^2}{c^2},$$

$$A_2 = 2k_y^2\beta^2(1-\omega^2/k^2c^2) + \frac{(\omega^+)^2}{c^2},$$

$$\Delta_e(\omega) = \frac{\varepsilon q_0 - q}{\varepsilon q_0 + q}, \quad \Delta_m(\omega) = \frac{\mu q_0 - q}{\mu q_0 + q}.$$

In the limit $V/c \ll 1$ ($\gamma = 1$), taking the retardation into account, the quantity dQ'/dt' coincides with the rate of particle heating dQ/dt in the reference frame of the plate. In this case, formula (S1) reduces to (1).

B. Structure of formula (1). In the case $V = 0$, formula (1) coincides with the well-known results [3, 6–8]. For $V \neq 0$, its key feature is the presence of the frequency factor ω^+ in the integrand. It is the factor ω^+ that mathematically leads to the possibility of anomalous heating of the particle. To illustrate the appearance of ω^+ in (5), we consider a simpler case of a nonretarded nonrelativistic interaction of a small particle with a fluctuating electric dipole moment. In this case, the initial expression for the particle heating rate dQ/dt is given by [13]

$$dQ/dt = dQ^{(1)}/dt + dQ^{(2)}/dt = \langle\dot{\mathbf{d}}^{\mathrm{sp}}\mathbf{E}^{\mathrm{ind}}\rangle + \langle\dot{\mathbf{d}}^{\mathrm{ind}}\mathbf{E}^{\mathrm{sp}}\rangle, \quad \text{(S2)}$$

where indices "sp" and "ind" denote spontaneous and induced components of the fluctuating dipole moment of the particle and the electric field of the surface, the dots over $\mathbf{d}$ denote time derivatives, and the angular brackets denote complete quantum statistical averaging. When calculating the first term in (S2), the solution to the Poisson equation $\Delta\phi = 4\pi\,\mathrm{div}\,\mathbf{P}$ for the electric potential ϕ has to be fond, where $\mathbf{P} = \delta(x - Vt)\delta(y)\delta(z - z_0)\mathbf{d}^{\mathrm{sp}}(t)$ is the polarization created by fluctuating dipole moment $\mathbf{d}^{\mathrm{sp}}(t)$ of a particle. The ϕ is expressed by the integral Fourier-transform

$$\phi(\mathbf{r}, z, t) = \frac{1}{(2\pi)^3}\int d\omega d^2k\phi(\omega,\mathbf{k};z)\exp(i(\mathbf{kr} - \omega t)), \quad \text{(S3)}$$

where $\mathbf{r} = (x, y)$, $\mathbf{k} = (k_x, k_y)$. The $\mathbf{d}^{\mathrm{sp}}(t)$ is expressed by

$$\mathbf{d}^{\mathrm{sp}}(t) = \frac{1}{(2\pi)}\int d\omega\mathbf{d}(\omega)\exp(-i\omega t). \quad \text{(S4)}$$

The Poisson equation is solved under the standard boundary conditions $\phi(\mathbf{r}, +0, t) = \phi(\mathbf{r}, -0, t)$, $\partial_z(\mathbf{r}, z, t)_{z=+0} = \varepsilon\partial_z(\mathbf{r}, z, t)_{z=-0}$, where ε is the dielectric permittivity of the plate. For the Fourier-component of the induced potential created by the moving particle, it follows [13]

$$\phi^{\mathrm{ind}}(\omega,\mathbf{k};z) = \frac{2\pi}{k}\Delta(\omega)\exp(-k(z+z_0))[i\mathbf{k}\mathbf{d}^{\mathrm{sp}}(\omega - k_xV) + kd_z^{\mathrm{sp}}(\omega - k_xV)], \quad \text{(S5)}$$

where $\Delta(\omega) = (\varepsilon(\omega) - 1)/(\varepsilon(\omega) + 1)$. Using (S5) and the relationship $\mathbf{E}^{\mathrm{ind}} = -\nabla\phi^{\mathrm{ind}}$, the induced electric field at the particle location point $(Vt, 0, z_0)$ is given by

$$\mathbf{E}^{\mathrm{ind}} = \frac{1}{(2\pi)^3}\int d\omega d^2k\phi^{\mathrm{ind}}(\omega, \mathbf{k}; z)\exp(-\mathrm{i}(\omega - k_x V)). \tag{S6}$$

Having substituted (S4) and (S6) into the first term of (S2) and taking the correlator of the particle dipole moments into account

$$\langle d_i^{\mathrm{sp}}(\omega)d_k^{\mathrm{sp}}(\omega')\rangle = 2\pi\delta_{ik}\hbar\delta(\omega + \omega')\alpha''(\alpha)\coth\frac{\hbar\omega}{2T_1}, \tag{S7}$$

where i, $k = x, y, z$ and $\alpha''(\omega)$ is the imaginary part of the particle polarizability, we obtain

$$\frac{dQ^{(1)}}{dt} = -\frac{\hbar}{\pi^2}\int\limits_0^{\infty} d\omega \int\limits_{-\infty}^{+\infty} dk_x \int\limits_{-\infty}^{+\infty} dk_y k\omega^+ e^{-2kz_0}\alpha''(\omega^+)\mathrm{Im}\Delta(\omega)\coth\frac{\hbar\omega^+}{2T_1}, \tag{S8}$$

where $\omega^+ = \omega + k_x V$. When obtaining (S8), the analytical properties of the functions $\alpha(\omega)$ and $\Delta(\omega)$ are used (evenness of their real parts and oddness of imaginary parts). When calculating the term $dQ^{(2)}/dt$ in (S2), the linear integral relation between $\mathbf{E}^{\mathrm{sp}}$ and $\mathbf{d}^{\mathrm{ind}}$ is used, yielding

$$\mathbf{d}^{\mathrm{ind}}(t) = \frac{1}{(2\pi)^3}\int d\omega\alpha(\omega - k_x V)\mathbf{E}^{\mathrm{sp}}(\omega, \mathbf{k}; z_0)\exp(-(\omega - k_x V)t). \tag{S9}$$

The correlator of electric fields of the plate arising in this case is worked out using the fluctuation-dissipation relation [13]

$$\langle\mathbf{E}^{\mathrm{sp}}(\omega, \mathbf{k}; z_0)\mathbf{E}^{\mathrm{sp}}(\omega', \mathbf{k}'; z_0)\rangle = 2(2\pi)^4\hbar k e^{-2kz_0}\Delta''(\omega)\delta(\omega + \omega')\delta(\mathbf{k} + \mathbf{k}')\coth\frac{\hbar\omega}{2T_2}. \tag{S10}$$

Substituting (S9) and (S10) into the second term of (S2) yields

$$\frac{dQ^{(2)}}{dt} = \frac{\hbar}{\pi^2}\int\limits_0^{\infty} d\omega \int\limits_{-\infty}^{+\infty} dk_x \int\limits_{-\infty}^{+\infty} dk_y k^+ e^{-2kz_0}\alpha''(\omega^+)\mathrm{Im}\Delta(\omega)\coth\frac{\hbar\omega}{2T_2}. \tag{S11}$$

Summing up (S9) and (S11), yields

$$\frac{dQ^{(2)}}{dt} = \frac{\hbar}{\pi^2}\int\limits_0^{\infty} d\omega \int\limits_{-\infty}^{+\infty} dk_x \int\limits_{-\infty}^{+\infty} dk_y k\omega^+ e^{-2kz_0}\alpha''(\omega^+)\mathrm{Im}\Delta(\omega)\left(\coth\frac{\hbar\omega}{2T_2} - \coth\frac{\hbar\omega^+}{2T_1}\right). \tag{S12}$$

Formula (S11) coincides with (1) when passing in the latter to the non-retarded limit ($c \to \infty$, $q_0 \to k$). In the relativistic solution to this problem, it follows $\Delta(\omega) \to (\varepsilon q_0 - q)/(\varepsilon q_0 + q)$. Similarly, if the heating of the particle due to the magnetic interaction is taken into account, one obtains $dQ/dt = \langle\dot{\mathbf{m}}^{\mathrm{sp}}\mathbf{B}^{\mathrm{ind}} + \langle\dot{\mathbf{m}}^{\mathrm{ind}}\mathbf{B}^{\mathrm{sp}}\rangle$, where $\mathbf{m}^{\mathrm{sp,ind}}$ и $\mathbf{B}^{\mathrm{sp,ind}}$ are the spontaneous and induced components of the fluctuation magnetic moment of the particle and the magnetic field of the plate.The corresponding calculations lead to formula (S11) with the electric polarizability replaced by the magnetic one and $\Delta(\omega) \to (\mu - 1)/(\mu + 1)$. Upon relativistic consideration, respectively, $\Delta(\omega) \to (\mu q_0 - q)/(\mu q_0 + q)$.

Thus, the appearance of a "shifted" frequency ω^+ in the formulas for the heating rate dQ/dt of a particle is mathematically due to the presence of derivatives of the dipole moment in (S2), which must be taken before substituting the instantaneous coordinates of the particle $(Vt, 0, z_0)$, with subsequent application of analytical properties of polarizability $\alpha(\omega)$ and the Fresnel reflection coefficients $\Delta(\omega)$ of plate.