Supplementary Material to the article

"Plasmons in stripe with anisotropic two-dimensional electron gas fully screened by metal gate"

1. The differential equation for the current

Let us consider the two-dimensional electron system presented in Fig. 1 of the article. We consider electromagnetic fields, density of current and charges proportional to $e^{iq_y y - i\omega t}$, where q_y is a real wavevector along the stripe and ω is a complex frequency. Using the Lorenz gauge for the electric potential φ and the vector potential $\mathbf{A}$

$$-i\frac{\omega}{c}\varphi(x,z) + \operatorname{div}\mathbf{A}(x,z) = 0, \tag{S1}$$

the Maxwell's equations in potential formulation are written as

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} - q_y^2 + \frac{\omega^2}{c^2}\right]\mathbf{A}(x,z) = -\frac{4\pi}{c}\mathbf{j}(x)\delta(z), \tag{S2}$$

where $\mathbf{j}(x)$ is the current density in the stripe, $\delta(z)$ is the Dirac delta function. Since the z-component of the current density is absent, there is no source for A_z and therefore $A_z = 0$. Then we apply the Fourier transform by the x-coordinate to (S2) and obtain the following ordinary differential equation

$$\left[\frac{\partial^2}{\partial z^2} - \beta^2\right]\mathbf{A}(q_x,z) = -\frac{4\pi}{c}\mathbf{j}(q_x)\delta(z), \tag{S3}$$

where $\beta^2 = q_x^2 + q_y^2 - \omega^2/c^2$. Further, we solve this equation taking into account that on the ideal metal gate tangential components of the electric field is equal to zero and, therefore, the vector potential vanishes on the metal. Besides, at $z > 0$ we choose the solution that vanishes at $z \to \infty$ and connect the vector potentials above and below the stripe plane using the condition

$$\frac{\partial}{\partial z}\mathbf{A}(q_x, z=+0) - \frac{\partial}{\partial z}\mathbf{A}(q_x, z=-0) = -\frac{4\pi}{c}\mathbf{j}(q_x) \tag{S4}$$

dictated by the presence of the Dirac delta function in the right side of the equation (S3). Thus, we get the following solution

$$\mathbf{A}(q_x,z) = \frac{2\pi}{c\beta}\mathbf{j}(q_x)\left(e^{-\beta|z|} - e^{-\beta(z+2d)}\right). \tag{S5}$$

Here $\operatorname{Re}\beta \geq 0$ that corresponds to the vanishing electromagnetic fields at $z \to \infty$. After applying the inverse Fourier transform the vector potential in the stripe plane is

$$\mathbf{A}(x) \equiv \mathbf{A}(x, z=0) = \frac{1}{c}\int_{-\frac{W}{2}}^{\frac{W}{2}} dx' G(x-x')\mathbf{j}(x'), \tag{S6}$$

with the kernel

$$G(x) = \int_{-\infty}^{\infty} dq_x \frac{e^{iq_x x}}{\beta}\left(1 - e^{-2\beta d}\right). \tag{S7}$$

Using the Lorenz gauge (S1) and the definition of the electric field through the electric and vector potentials

$$\mathbf{E} = -\operatorname{grad}\varphi - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{A}, \tag{S8}$$

we find the connection between the current density $\mathbf{j}$ and the induced electric field $\mathbf{E}$ in the stripe

$$\mathbf{E}(x) = \frac{i}{\omega}\begin{pmatrix} \frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2} & iq_y\frac{\partial}{\partial x} \\ iq_y\frac{\partial}{\partial x} & -q_y^2 + \frac{\omega^2}{c^2}\end{pmatrix}\int_{-\frac{W}{2}}^{\frac{W}{2}} dx' G(x-x')\mathbf{j}(x'). \tag{S9}$$

To obtain the general integro-differential equation for the current density, we use the local Ohm's law $\mathbf{j} = \sigma \mathbf{E}$ with conductivity tensor σ.

Assuming $\beta d \ll 1$ we expand $1 - e^{-2\beta d}$ as $2\beta d$ and then the integral kernel $G(\xi)$ reduces to $4\pi d \delta(\xi)$. Thus, we obtain the differential Eq. (2) discussed in the main text

$$\mathbf{j}(x) = i\frac{4\pi d\sigma}{\omega}\begin{pmatrix} \frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2} & iq_y \frac{\partial}{\partial x} \\ iq_y \frac{\partial}{\partial x} & -q_y^2 + \frac{\omega^2}{c^2} \end{pmatrix}\mathbf{j}(x). \tag{S10}$$

Bellow we find the solution of the equation with boundary conditions $j_x(-W/2) = j_x(W/2) = 0$.

2. Plasma eigenmodes. From the second equation of the system (S10) we express y-component of the current density

$$\left[\left(\sigma^{-1}\right)_{yy} + i\frac{4\pi d}{\omega}\left(q_y^2 - \frac{\omega^2}{c^2}\right)\right] j_y(x) = -\frac{4\pi d q_y}{\omega}\frac{\partial j_x(x)}{\partial x} - \left(\sigma^{-1}\right)_{yx} j_x(x). \tag{S11}$$

Now we have to consider two different cases. The first one is realized when the condition

$$\left(\sigma^{-1}\right)_{yy} + i\frac{4\pi d}{\omega}\left(q_y^2 - \frac{\omega^2}{c^2}\right) = 0 \tag{S12}$$

is satisfied. In this case, the current has the following form

$$j_x(x) = 0, \quad j_y(x) = j_y(0) e^{-\frac{\omega\left(\sigma^{-1}\right)_{xy}}{4\pi d q_y}x}. \tag{S13}$$

Using equation (S12), the conductivity tensor (1) from the article and the condition $|\omega| \gg \gamma$ we find the plasma frequency and damping written in Eqs. (3) and (4) in the article respectively. This mode corresponds to the edge magnetoplasmon.

In the second case, the condition (S12) is not fulfilled, therefore we express the y-component of the current in terms of the x-component, substitute it into the first equation of the system (S10), and obtain the following differential equation:

$$\frac{\partial^2 j_x(x)}{\partial x^2} + 2ik_1\frac{\partial j_x(x)}{\partial x} + k_2^2 j_x(x) = 0, \tag{S14}$$

where, for the sake of brevity, we introduce

$$k_1 = \frac{q_y\left(\sigma_{xy} + \sigma_{yx}\right)}{2\left(\sigma_{xx} - i\frac{4\pi d\omega}{c^2}\det\sigma\right)}, \qquad \det\sigma = \sigma_{xx}\sigma_{yy} - \sigma_{xy}\sigma_{yx},$$

and

$$k_2^2 = \frac{\omega^2}{c^2} + \frac{i\frac{4\pi d\omega q_y^2}{c^2}\det\sigma - \left(q_y^2 - \frac{\omega^2}{c^2}\right)\sigma_{yy} + i\frac{\omega}{4\pi d}}{\sigma_{xx} - i\frac{4\pi d\omega}{c^2}\det\sigma}. \tag{S15}$$

We search the solution of equation (S14) as a superposition of plane waves having a form $e^{iq_x x}$ and obtain the following characteristic equation:

$$q_x^2 + 2k_1 q_x - k_2^2 = 0. \tag{S16}$$

Consequently, the general form of the current is

$$j_x(x) = D_1 e^{iq_{x+}x} + D_2 e^{iq_{x-}x}, \tag{S17}$$

where

$$q_{x\pm} = -k_1 \pm \sqrt{k_1^2 + k_2^2}, \tag{S18}$$

D_1 and D_2 are constant coefficients. Using the boundary condition we find the solution

$$j_x(x) = Ce^{-ik_1 x}\sin\left(n\frac{\pi x}{W} - n\frac{\pi}{2}\right), \tag{S19}$$

$$j_y(x) = e^{-ik_1 x}\left[C_1\sin\left(n\frac{\pi x}{W} - n\frac{\pi}{2}\right) + C_2\cos\left(n\frac{\pi x}{W} - n\frac{\pi}{2}\right)\right], \tag{S20}$$

where

$$C_1 = C\frac{i\frac{4\pi d q_y k_1}{\omega} + \frac{\sigma_{yx}}{\det\sigma}}{\frac{\sigma_{xx}}{\det\sigma} + i\frac{4\pi d}{\omega}\left(q_y^2 - \frac{\omega^2}{c^2}\right)}, \tag{S21}$$

$$C_2 = -C\frac{\frac{4\pi d q_y}{\omega}\frac{n\pi}{W}}{\frac{\sigma_{xx}}{\det\sigma} + i\frac{4\pi d}{\omega}\left(q_y^2 - \frac{\omega^2}{c^2}\right)}, \tag{S22}$$

and C, n are any complex and natural number respectively. The nontrivial solution of the equation (S14) is defined by $q_{x+} - q_{x-} = 2n\pi/W$, which is the dispersion equation of plasma modes designated as "bulk" in the article. Note, that it results in $q_{x\pm} = k_1 \pm n\pi/W$ with $k_1 = 0$ only in isotropic system. Therefore, in anisotropic system we have $q_{x+} \neq -q_{x-}$. Using the derived dispersion equation and the conductivity (1), we get the frequency $\mathrm{Re}\,\omega_n(q_y, \theta)$ of the plasma resonances when $\gamma = 0$, and the plasma damping $\mathrm{Im}\,\omega_n(q_y, \theta)$ in the first order by the electron collisional damping rate γ.