Supplementary Material to the article
«Optimization of adiabatic superconducting logic cells by Josephson π-junctions»

Evaluating the effect of π-junctions

The addition of π-junctions to the parametric quantron (PQ) makes it possible to reduce the requirements for the design of the logic cell. As an example, the "negative SQUID" (n-SQUID) and the Adiabatic Quantum Flux Parametron (AQFP) are considered below, with parameters taken from [1, 2].

In a variant of n-SQUID [2] clocking by an external alternating current is described, in which the magnetic flux is set in the inductance of the main circuit of the PQ, l_q. The parameters of the real cell are: the critical current of the Josephson junctions, I_c = 15.7 μA, the inductance of the, L = 40 pH, the inductance of the PQ main circuit, L_q = 1.7 pH, the negative mutual inductance between the SQUID shoulders, M = 16 pH. The normalised values of the inductances were: m = 0.76, l = 0.95, l_q = 0.08.

The critical current of the n-SQUID contacts I_c was about three times lower than the minimum critical current commonly used in superconducting logic circuits. In practice, the limitation on the minimum value of I_c is due to the most convenient operating temperature, T = 4.2 K, which corresponds to the boiling point of helium. The noise current at this temperature is of the order of $I_T \approx 18$ μA (formula for calculating the noise current: $I_T = (2\pi/\Phi_0)k_B T$), and as I_c decreases below the empirical threshold (~ 50 μA), the noise begins to have a significant impact on the operation of the circuits. To avoid errors caused by thermal fluctuations, the temperature in the experiment was lowered to 1.6 K.

Such a low value of the critical current was chosen so that the normalized values of the inductance ensured the possibility of an adiabatic and unambiguous evolution of the PQ state. In order to implement such an evolution, it is necessary that no intermediate states with three wells are formed during the transition from the one-well to the two-well form of the potential. Otherwise, there is a possibility of an error in the transmission of information. Due to thermal fluctuations, it is possible to change the sign of the phase associated with the logical state with respect to the phase associated with the cell input. The potential energy calculated by formula (11) from the main text of the article, at $i_{c3} = i_{c4} = 0$ for different values of the leading phase φ_x, corresponding to the unambiguous dynamics of the n-SQUID in Fig. S1. The system parameters for the calculation are taken from the experimental works mentioned above.

For example, increasing the critical current of the SQUID's Josephson contacts by a factor of 3 to a value close to the threshold, $I_c \approx 47.1$ μA, one leads to a corresponding increase in the normalised inductances: m = 2.17, l = 2.85, l_q= 0.24. However, the dynamics of an n-SQUID with such parameters will no longer be unambiguous, see Fig. S2. In a sufficiently large range of the leading phase, three potential wells appear.

This problem can be solved by correcting the Josephson term in the potential energy. Using a PQ with two π-contacts (Fig. 9 from the main text of the article) with critical currents $i_{c3} = 0.5, i_{c4} = 0.75$, we pass from the original Josephson relief (Fig. S3a) to the relief shown in Fig. S3c, due to the "corrective" term corresponding to the energy of the π-contacts (Fig. S3b). Corresponding to the change in the Josephson relief, the dynamics of the n-SQUID will now be unambiguous (Fig. S4), allowing operation at a standard temperature of 4.2 K.

The system of equations (4) from the main text of the article was used for the calculation. The same dependences for n-SQUID with parameters corresponding to a threefold increase in the critical currents of Josephson junctions are shown by dotted lines in the corresponding figures. Since parts of the dependences with a negative slope of the derivatives $\frac{d\varphi_-}{d\varphi_x}$ and $\frac{d(\varphi_+ - \varphi_x)}{d\varphi_x}$ are not realised in the experiment, the dynamics of such a cell cannot be adiabatic and is accompanied by phase jumps.

References

1. N. Takeuchi, K. Ehara, K. Inoue, Y. Yamanashi, IEEE Trans. Appl. Supercond. 23, 1700304 (2013).
2. H. Li, J. Liu, Y. Zhang, H. Cai, G. Li, Q. Liu, S. Han, W. Chen, Supercond. Sci. Technol., 30, 035012 (2017).

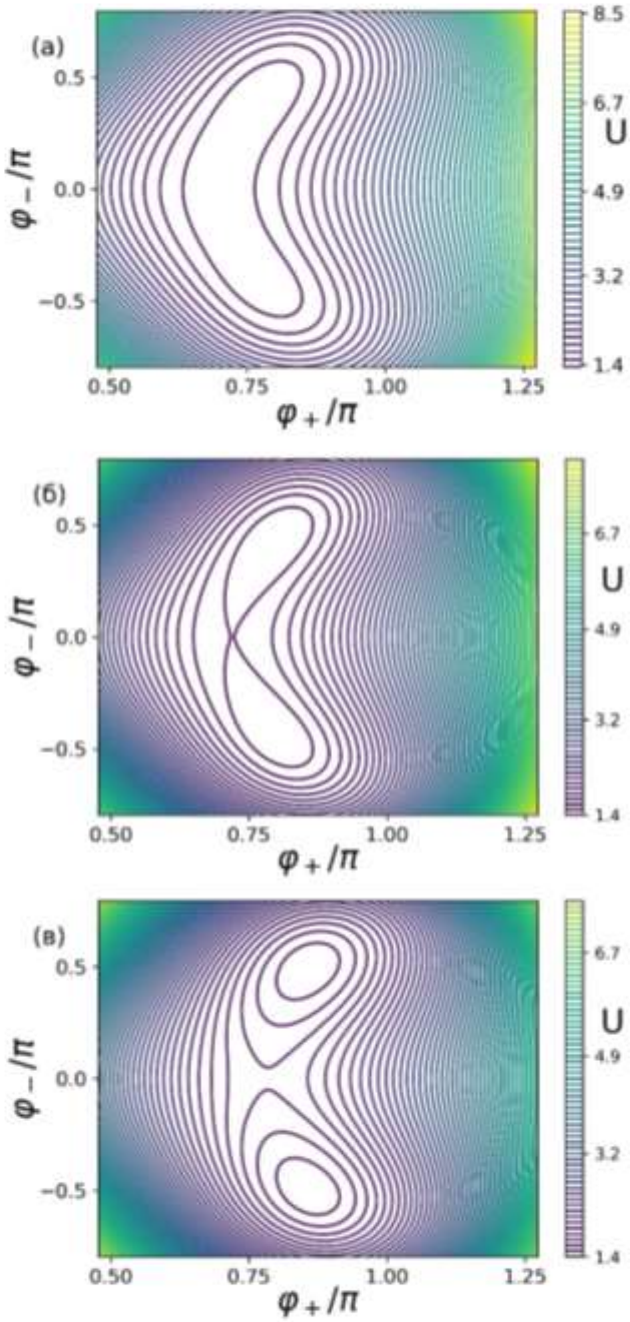

Fig. S1. Potential energy of the n-SQUID with parameters taken from [25]. The energy (here and below normalized by the Josephson energy, E_J) is calculated for phases (see Fig. 3): φ_t = 0 и φ_x = 2.5 (a), φ_x = 2.55 (b), φ_x = 2.7 (c).

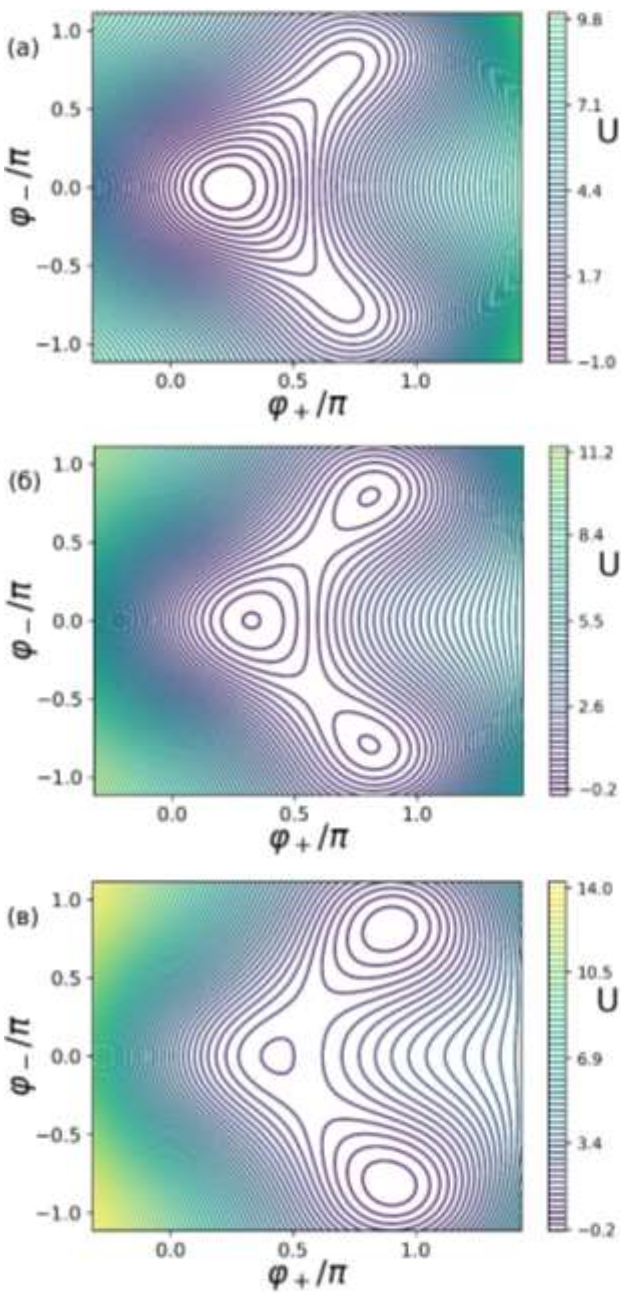

Fig. S2. Potential energy of the n-SQUID with normalised inductance values three times higher than the "experimental" values: m = 2.17, l = 2.85, l_q= 0.24. The calculation is given for phases (see Fig. 3): φ_t = 0 и φ_x = 1.5 (a); φ_x = 2 (b); φ_x = 2.5 (c).

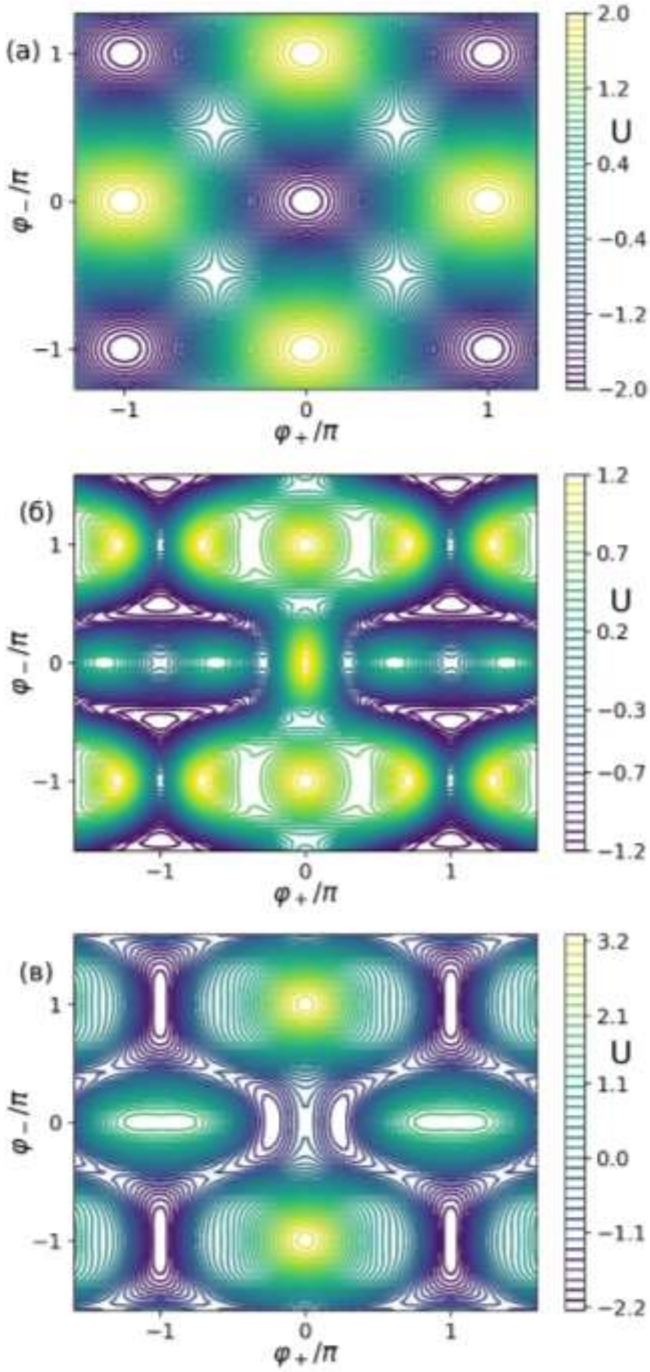

Fig. S3. Josephson relief in the potential energy of the n-SQUID (a); relief corresponding to the energy of π-contacts in (11) (b); relief for the PQ with π-contacts (Fig. 9) (c). Parameter values: m = 2.17, l = 2.85, l_q= 0.24, i_{c3} = 0.5, i_{c4} = 0.75.

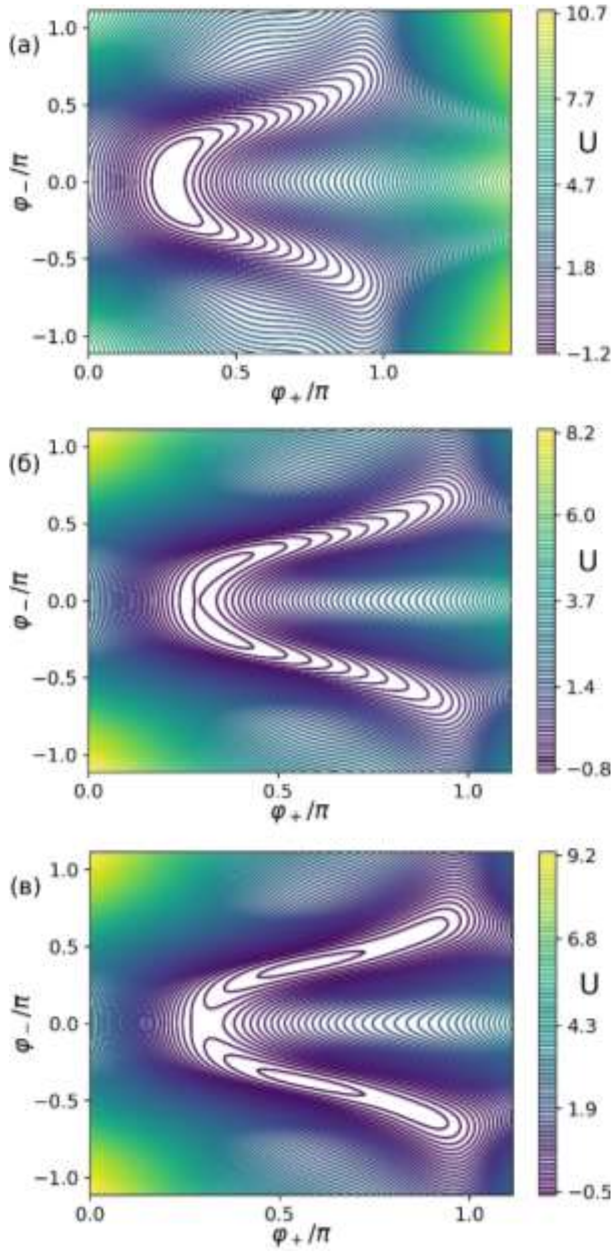

Fig. S4. Potential energy of the PQ with π-contacts with parameters: m = 2.17, l = 2.85, l_q= 0.24, i_{c3} = 0.5, i_{c4} = 0.75 for phase values (see Fig. 9): φ_t = 0, и φ_x = 1.5 (a), φ_x = 1.8 (b), φ_x = 2.1 (c).