Supplementary material for High-frequency Hall effect and Transverse Electric galvanomagnetic waves in current-biased 2d electron systems

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1 On absorption of circularly polarized light by 2DES and its chirality

Here we examine the problem of a circularly polarized light incident on a 2DES with tensor-valued conductivity $\hat{\sigma}$ (Fig. 1). The polarization (right or left) of incident radiation is encoded in parameter p. The straightforward solution of Maxwell's equations with corresponding boundary conditions leads us to the following values for the magnitudes of reflection and transmission coefficients:

$$t_+ = \frac{1}{2}\frac{\Sigma_{xx}+\Sigma_{yy}+2+ip\left(\Sigma_{yx}-\Sigma_{xy}\right)}{\left(\Sigma_{xx}+1\right)\left(\Sigma_{yy}+1\right)-\Sigma_{xy}\Sigma_{yx}}; \tag{1}$$

$$t_- = \frac{1}{2}\frac{\Sigma_{yy}-\Sigma_{xx}-ip\left(\Sigma_{xy}+\Sigma_{yx}\right)}{\left(\Sigma_{xx}+1\right)\left(\Sigma_{yy}+1\right)-\Sigma_{xy}\Sigma_{yx}}; \tag{2}$$

$$r_+ = t_-; \quad r_- = t_+ - 1. \tag{3}$$

The absorption coefficient α is given by

$$\alpha = 1 - |r_+|^2 - |r_-|^2 - |t_+|^2 - |t_-|^2. \tag{4}$$

We are interested in polarization-sensitive terms, so we examine each of the modulus-squared terms in the above equation:

$$|r_+|^2 = |t_-|^2 \propto p^2\left(\Sigma''_{xy}+\Sigma''_{yx}\right)^2 + \left(\Sigma''_{yy}-\Sigma''_{xx}\right)^2; \tag{5}$$

$$|t_+|^2 \propto \left[2 - p\left(\Sigma''_{yx}-\Sigma''_{xy}\right)\right]^2 + \left(\Sigma''_{xx}+\Sigma''_{yy}+2\right)^2; \tag{6}$$

$$|r_-|^2 \propto \left[\Sigma''_{xx}\Sigma''_{yy} - \Sigma''_{xy}\Sigma''_{yx} - p\left(\Sigma''_{yx}-\Sigma''_{xy}\right)\right]^2 + \left(\frac{\Sigma''_{xx}+\Sigma''_{yy}}{2}\right)^2, \tag{7}$$

where the imaginary part of any quantity χ is denoted as χ''.

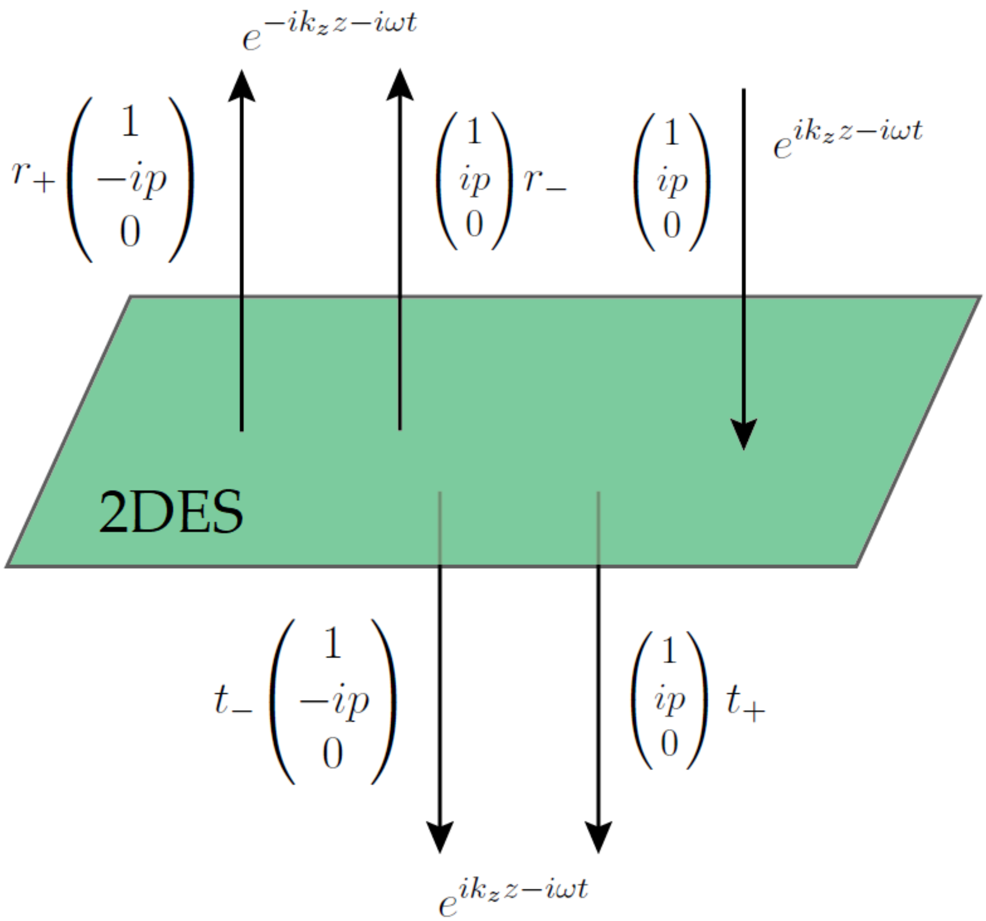

Figure 1: Schematic drawing of a circularly polarized wave incident on a 2DES in vacuum. The 2DES is characterized by a conductivity tensor $\hat{\sigma}$.

Thus, we conclude that the absorption coefficient contains the following polarization-sensitive term:

$$-p\left(\Sigma''_{yx}-\Sigma''_{xy}\right)\frac{1+\Sigma''_{xx}\Sigma''_{yy}-\Sigma''_{xy}\Sigma''_{yx}}{\left|\left(\Sigma_{xx}+1\right)\left(\Sigma_{yy}+1\right)-\Sigma_{xy}\Sigma_{yx}\right|^{2}}\in\alpha. \tag{8}$$

From this we conclude that a 2DES with non-symmetric conductivity tensor ($\Sigma_{xy}\neq\Sigma_{yx}$) absorbs light dependent on its polarization, i.e. such a system is *chiral*. This is a usual situation for 2DESs in external magnetic field, but obviously cannot be attributed to 2DESs with homogeneously drifting carriers.

2 Solution to dispersion relation $\det\hat{M}_2 = 0$ when drift and wave vector are not parallel

After tedious algebra one can obtain the general form of the dispersion relation $\det\hat{M}_2 = 0$ (Eq. (16) of the main text)

$$\Sigma_{xx}\Sigma_{yy} + i\Sigma_{xx}\left(-\frac{\sin^2\alpha}{\kappa_z} + \kappa_z\cos^2\alpha\right) + i\Sigma_{yy}\left(-\frac{\cos^2\alpha}{\kappa_z} + \kappa_z\sin^2\alpha\right) - \\ - \Sigma_{xy}\Sigma_{yx} + \left(\Sigma_{xy} + \Sigma_{yx}\right)\frac{i\kappa^2}{2\kappa_z}\sin 2\alpha + 1 = 0. \tag{9}$$

This equation can be simplified in the quasistatic limit ($\kappa_z \simeq \kappa$) to

$$1 - \frac{\Sigma_0^2}{\left(1 + i/\omega\tau - \kappa\beta\cos\alpha\right)^2} + \Sigma_0\frac{(\kappa\beta)^2(3+\cos 4\alpha)/4 - \kappa^2 + 1 - 2\kappa\beta\cos^3\alpha}{\kappa\left(1 - \kappa\beta\cos\alpha\right)\left(1 + i/\omega\tau - \kappa\beta\cos\alpha\right)} = 0, \tag{10}$$

where $\beta = u_0/c$. This equation can be solved analytically with respect to κ. In the most relevant limit $\beta \ll 1$, $\kappa \gg \omega, \Sigma_0$ (or $u_0 \ll c$, $kc \gg \omega, \omega_{2d}$) we come to

$$1 - \frac{\Sigma_0^2}{\left(1 + i/\omega\tau - \kappa\beta\cos\alpha\right)^2} - \frac{\kappa\Sigma_0}{\left(1 - \kappa\beta\cos\alpha\right)\left(1 + i/\omega\tau - \kappa\beta\cos\alpha\right)} = 0. \tag{11}$$

This is a cubic equation with respect to ω and can be solved analytically. One of its roots gives the dispersion (21) of the main text.