Supplementary Material to the article "The Effect of Spatial Correlations of Impurity Magnetization on the Transition Temperature to the A_1-phase of Superfluid ^{3}He"

1.

As it was previously mentioned in the main body of the article, in the simplest approximation the Abrikosov-Gor'kov theory does not take into account the spatial inhomogeneity of the order parameter. The influence of the inhomogeneity of the order parameter on the shift of transition temperature T_c for triplet pairing within the framework of this theory was considered in [1]. However, the result obtained in that work is too cumbersome and its analysis is rather difficult. Since in the vicinity of transition temperature the self-consistency Gor'kov equation reduces to the Ginzburg-Landau equation, it seems possible to perform all calculations in this limit. In order to do this, it is needed that the inhomogeneity of the aerogel density reveals on scales significantly exceeding the coherence length of superfluid ^{3}He. The most interesting situation is that of low impurity concentrations, where the shift of T_c is small compared to the unperturbed problem ($\frac{\xi_0}{l_{tr}} \ll 1$). In this case, a term that depends on spatial coordinates and describes the effective interaction of the superfluid system with magnetic impurities can be added to the free energy density. Below is the derivation of this term, based on the approximation of classical trajectory [2].

Let us write down the density of Ginzburg-Landau free energy in the form:

$$F^{(2)} = \frac{1}{2\lambda}\int d^3r A^*_{\mu i}(\mathbf{r})A_{\mu i}(\mathbf{r}) - \int d^3r_1 d^3r_2 A^*_{\mu i}(\mathbf{r}_1)K_{ij,\mu\nu}(\mathbf{r}_1,\mathbf{r}_2)A_{\nu j}(\mathbf{r}_2), \quad \text{(S1)}$$

where $A_{\mu j}$ – the order parameter of superfluid ^{3}He, and λ – an interaction constant. The key result of [2] is the possibility to express the kernel of the equation $K_{ij,\mu\nu}(\mathbf{r}_1,\mathbf{r}_2)$ through the classical current-current correlator:

$$K_{il,\mu\nu}(\mathbf{r}_1,\mathbf{r}_2) = \frac{6\pi N_{eff}T}{k_F^2}\cdot \sum_n \int_0^\infty dt e^{-2|\omega_n|t}\langle j^\mu_l(\mathbf{r}_1,t=0)j^\nu_p(\mathbf{r}_2,t)\rangle_{classic}, \quad \text{(S2)}$$

where N_{eff} is the effective density of states at the Fermi surface, $\omega_n = (2n-1)\pi T$ – Matsubara frequencies, $n = 0,\pm1,\pm2,...$ The expression for the classical correlator has the form:

$$\langle j^\mu_i(\mathbf{r}_1,t=0)j^\nu_l(\mathbf{r}_2,t)\rangle_{classic} = \int \frac{d\Omega_1}{4\pi}\frac{d\Omega_2}{4\pi}\frac{(p_1)_i(p_2)_l}{(M^*)^2}\cdot\delta(\mathbf{r}_1+\Delta\mathbf{r}(\mathbf{p}_1,\mathbf{p}_2,t)-\mathbf{r}_2) (\sigma_y\sigma_\mu)_{\alpha\beta}w(\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2)_{\alpha\gamma\beta\zeta}(\sigma_\nu\sigma_y)_{\gamma\zeta}, \quad \text{(S3)}$$

where σ_i are the Pauli matrices, M^* is the effective mass of a quasiparticle, $\Omega_{1,2}$ – solid angle in the direction $\mathbf{p}_{1,2}$ correspondingly ($|\mathbf{p}_{1,2}| = p_F$), $\mathbf{r}_1 + \Delta\mathbf{r}(\mathbf{p}_1,\mathbf{p}_2,t)$ is quasiparticle position after time t for the given trajectory, $w(\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2)_{\alpha\gamma\beta\zeta}$ is the matrix, that defines the scattering probability. For the direct trajectories connecting two poins without any act of scattering the probability matrix can be represented as:

$$w(\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2)_{\alpha\gamma\beta\zeta} = 4\pi\delta(\hat{\mathbf{p}}_1-\hat{\mathbf{p}}_2)(\sigma_0)_{\alpha\gamma}\cdot(\sigma_0)_{\beta\zeta}, \quad \text{(S4)}$$

σ_0 – 2×2 unit matrix and

$$\delta(\mathbf{r}_1+\frac{\mathbf{p}_1}{M^*}t-\mathbf{r}_2) = \frac{\delta(\frac{\mathbf{r}_2-\mathbf{r}_1}{|\mathbf{r}_2-\mathbf{r}_1|}-\frac{\mathbf{p}_1}{p_1})}{|\mathbf{r}_2-\mathbf{r}_1|^2}\delta(|\mathbf{r}_2-\mathbf{r}_1|-v_Ft).$$

Using the written above expressions it is easy to arrive at the standard form of the bulk part of the kernel of unperturbed problem. Here we need to make a reservation: the triviality of spin structure of the kernel for direct trajectories is the consequence of equality of the density of states for spin projections $\pm\frac{1}{2}$. If the system is placed in an external magnetic field directed along the z-axis, then when taking the trace of $(\sigma_z)_{\alpha\beta}$ it is necessary to account for the difference in the density of states for the spin projections $+1/2$ and $-1/2$. From the explicit form of the spin part of the correlator, it follows that:

$$\frac{1}{2N_F}Tr\left(N_F(\alpha)(\sigma_y\sigma_\mu)_{\alpha\beta}(\sigma_y\sigma_\nu)_{\beta\varphi}\right) \approx \delta_{\mu\nu} - ie_{\mu\nu z}\frac{N_F'}{N_F}\mu_B H, \quad \text{(S5)}$$

where we have used the property $\sigma_y\sigma_z\sigma_y = -\sigma_z$, and introduced the notation N_F' – the derivative of the density of states with respect to energy at the Fermi surface. It is the second term, followed from the

particle-hole asymmetry, that leads to the splitting of the transition temperature for opposite spin projections of the Cooper pair. It is worth noting, that the effect of the magnetic field is not limited to this term. In addition to it, there is a quadratic term in the magnetic field, describing the change in the magnetic susceptibility of the superfluid state. After integration over time in the expression for the kernel, the dominant term in the series over Matsubara frequencies for direct trajectories will be: $e^{-\frac{2\pi T|\mathbf{r}_2-\mathbf{r}_1|}{v_F}} \sim e^{-\frac{|\mathbf{r}_2-\mathbf{r}_1|}{\xi_0}}$, i.e. the kernel decays at distance $\xi_0 \equiv \frac{\hbar v_F}{2\pi T_c}$. This, in turn, allows us to neglect trajectories with double or more scatterings of quasiparticles on impurities, since the mean free path between two collisions in aerogels l_{tr} is much greater than the coherence length ξ_0.

Now, let us consider the model of scattering on a single impurity and calculate the resulting contribution to the kernel. The probability of classical elastic scattering with a change in the direction of momentum is a function of the scalar products $\frac{1}{p_F}(\mathbf{p}_{1,2}\mathbf{n})$, where $\mathbf{n}$ is the vector of the external normal to the scattering surface, $\mathbf{p}_1$ is the momentum of the quasiparticle before reflection, $\mathbf{p}_2$ is the momentum after reflection. The scattering matrix on a surface element with a given normal vector $\mathbf{n}$ and averaged spin $\langle\delta S_\varphi(\mathbf{n})\rangle$ can be written in the most general form as:

$$\begin{aligned} w_{\alpha\gamma\beta\zeta}(\mathbf{p}_1,\mathbf{p}_2;\mathbf{n}) = w_0(\mathbf{p}_1,\mathbf{p}_2;\mathbf{n})\cdot(\sigma_0)_{\alpha\gamma}\cdot(\sigma_0)_{\beta\zeta} + \\ w_1(\mathbf{p}_1,\mathbf{p}_2;\mathbf{n})\cdot\{(\sigma_0)_{\alpha\gamma}(\sigma_\varphi)_{\beta\zeta} + \\ (\sigma_\varphi)_{\alpha\gamma}(\sigma_0)_{\beta\zeta}\}\langle\delta S_\varphi(\mathbf{n})\rangle + \\ w_2(\mathbf{p}_1,\mathbf{p}_2;\mathbf{n})\cdot(\sigma_\varphi)_{\alpha\gamma}(\sigma_\omega)_{\beta\zeta}\langle\delta S_\varphi(\mathbf{n})\rangle\langle\delta S_\omega(\mathbf{n})\rangle, \end{aligned} \tag{S6}$$

$w_1 \sim J$, $w_2 \sim J^2$, J is the exchange integral, whereby $J > 0$ corresponds to antiferromagnetic exchange. After substitution of the expression for the probability into the correlator and summation over all trajectories connecting the given two points through the impurity surface, it is easy to obtain the spin structure of the contribution to the kernel from a single impurity:

$$\begin{aligned} \kappa_{ij,\mu\nu}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_a) = \delta_{\mu\nu}\cdot\kappa_{ij}^{(0)}(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a) + \\ +2ie_{\mu\nu\gamma}\langle S_\gamma^{(a)}\rangle\kappa_{ij}^{(1,a)}(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a) + \\ +\left(\delta_{\mu\nu}\langle S_\varphi^a S_\varphi^a\rangle - 2\langle S_\mu^a S_\nu^a\rangle\right)\kappa_{ij}^{(2,a)}(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a) \end{aligned} \tag{S7}$$

where $\mathbf{r}_a$ is the coordinate of the impurity center, $\langle S_\varphi^{(a)}\rangle(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a)$ is the spin of the impurity averaged over the scattering surface (accounting for the weight of the trajectories), $\kappa_{ij}^{(0),(1),(2)}(\mathbf{r}_1,\mathbf{r}_2)$ are the kernels calculated using probabilities w_0, w_1, w_2 correspondingly. Here, it is implicitly assumed that by specifying the coordinate of the impurity center, we can fully determine all possible trajectories. This is only true for spherically symmetric impurities. It should be emphasized that $\kappa_{ij}^{(0)}(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a)$ is related to the shape of the impurity and the type of quasiparticle scattering (diffusive or specular), while $\kappa_{ij}^{(1,a)}(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a)$ additionally depends on the spin distribution over the impurity surface and its appearance is, in fact, a consequence of the interference between potential scattering and exchange contribution to scattering. Finally, $\kappa_{ij}^{(2,a)}(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a) \sim J^2$ describes the contribution purely from the exchange interaction between the scattered quasiparticle and the surface. Furthermore, we note that the classical impurities not only add trajectories in the system but also affect direct trajectories due to the shadowing effect. The form of the tensor $\kappa_{ij}^{(0)}(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a)$ for distances $|\mathbf{r}-\mathbf{r}_a| \gg a$ was obtained in the work [3].

Let us separate in the kernel the part related to the direct trajectories of the system, which would exist in a pure system – $K_{ij,\mu\nu}^{bulk}$ and the contribution from impurity trajectories $K_{ij,\mu\nu}^{imp}$, i.e. $K_{ij,\mu\nu} = K_{ij,\mu\nu}^{bulk} + K_{ij,\mu\nu}^{imp}$. Additionally, the contribution to the kernel from a single impurity can be calculated independently. This is due to the fact that one can neglect multiple scatterings on impurities because of fast decay of the kernel with increasing trajectory length. Therefore: $K_{ij,\mu\nu}^{imp}(\mathbf{r}_1,\mathbf{r}_2) = \sum\limits_a \kappa_{ij,\mu\nu}^{(a)}(\mathbf{r}_1-\mathbf{r}_a,\mathbf{r}_2-\mathbf{r}_a)$. Next, we assume that the order parameter changes over distances much greater than ξ_0. In this case, due to the fast decay of the kernel as a function of $r = |\mathbf{r}_2-\mathbf{r}_1|$, one can expand the order parameter in the integral (S1), making the substitution ($\mathbf{r}_1 = \mathbf{R} - \frac{1}{2}\mathbf{r}$, $\mathbf{r}_2 = \mathbf{R} + \frac{1}{2}\mathbf{r}$):

$$\begin{aligned} A_{\mu j}(\mathbf{r}_1) = A_{\mu j}(\mathbf{R}) - \frac{r_i}{2}\nabla_i A_{\mu j}(\mathbf{R}) + \\ \frac{1}{8}r_i r_k \nabla_i\nabla_k A_{\mu j}(\mathbf{R}), \end{aligned} \tag{S8}$$

and integrate over $\mathbf{r}$. Due to the symmetry of the function $K_{ij,\mu\nu}^{bulk}(\mathbf{r}_1,\mathbf{r}_2)$ with respect to the permutation of $\mathbf{r}_1$, $\mathbf{r}_2$ the integrals linear in the gradient do not contribute to $F^{(2)}$. The quadratic terms in the gradient, after integration with $K_{ij,\mu\nu}^{bulk}(\mathbf{r}_1,\mathbf{r}_2)$ yields the standard gradient terms of the Ginzburg-Landau free energy density. As a result, the contribution to the energy density from the term with $K_{ij,\mu\nu}^{bulk}(\mathbf{r}_1,\mathbf{r}_2)$ can be written as:

$$\begin{aligned} N_F\left(\ln\frac{T}{T_c}\delta_{\mu\nu}\delta_{ij} - i\eta e_{\mu\nu\varphi}H_\varphi\delta_{ij}\right)A_{\mu i}A_{\nu j}^* + \\ +\frac{3}{5}N_F\xi_s^2\delta_{\mu\nu}\left(\nabla_i A_{\mu i}\nabla_j A_{\nu j}^* + \right. \\ \left. +\nabla_i A_{\mu j}\nabla_i A_{\nu j}^* + \nabla_i A_{\mu j}\nabla_j A_{\nu i}^*\right), \end{aligned} \tag{S9}$$

where $T_c = \hbar\omega_c e^{-\frac{1}{N_F\lambda}}$, $\eta = \frac{N_F^{'}\mu_B}{N_F\lambda}$, $\xi_s^2 = \frac{7\zeta(3)}{12}\xi_0^2$. The impurity contribution to the kernel affects both the homogeneous terms in the Ginzburg-Landau expansion and the gradient terms, making it possible to write a linear in gradients term proportional to the expression $A_i\nabla_k A_j^* - A_j^*\nabla_k A_i$. Further, we will restrict ourselves only to the contribution from impurities to the homogeneous term. After integration over one of the spatial coordinates, the following additional terms appears in the energy density:

$$N_F\left[\delta_{\mu\nu}\sum_a u_{ij}^{(0)}(\mathbf{r}-\mathbf{r}_a) + ie_{\mu\nu\varphi}\sum_a S_{\varphi,ij}^{(1,a)}(\mathbf{r}-\mathbf{r}_a) + \right.$$
$$\left. + \sum_a\left\{\delta_{\mu\nu}S_{\varphi\varphi,ij}^{(2,a)}(\mathbf{r}-\mathbf{r}_a) - 2S_{\mu\nu,ij}^{(2,a)}(\mathbf{r}-\mathbf{r}_a)\right\}\right]\times \quad \text{(S10)}$$
$$\times A_{\mu i}(\mathbf{r})A_{\nu j}^*(\mathbf{r})$$

where $u_{ij}^{(0)}(\mathbf{r}-\mathbf{r}_a)$ is the effective potential of a single impurity related to potential scattering, $S_{\varphi,ij}^{(1,a)}(\mathbf{r}-\mathbf{r}_a)$ is the potential accounting for the interference between potential and exchange interactions, and $S_{\mu\nu,ij}^{(2,a)}(\mathbf{r}-\mathbf{r}_a)$ is the symmetric potential with respect to spin indices. As can be seen from the definitions of the effective potentials, the term $S_{\varphi,ij}^{(1,a)}(\mathbf{r}-\mathbf{r}_a)$ is linear in the exchange integral, while the term $S_{\mu\nu,ij}^{(2,a)}(\mathbf{r}-\mathbf{r}_a)$ is quadratic. From the general property of the kernel, it follows that all three one-particle potentials decay at a distance ξ_0. As it was noted above, a more accurate *equation* should contain a linear in gradient term of the order parameter of the form $N_F\delta V_{\mu\nu,ij,k}^{(1)}(\mathbf{r})\nabla_k A_{\mu i}$, as well as a correction to the quadratic in gradient term $N_F\delta V_{\mu\nu,ij,kp}^{(2)}(\mathbf{r})\nabla_k\nabla_p A_{\mu i}$. The second term, in particular, describes a small renormalization of the coherence length, which is usually neglected. However, as it is seen from the logic of the article, the contribution of these two terms to the change in the transition temperature can be neglected due to the assumed small parameter $\xi_0/R \ll 1$.

2.

The solution of the equation for the fluctuating part of the order parameter $\chi_{\alpha\varphi,il}(\mathbf{r})$ is expressed by means of the Green's function which is defined as the solution of the equation:

$$\left((\hat{\Lambda}^{(0)T})_{\alpha\beta,ij}(\mathbf{r},\nabla_{\mathbf{r}},\tau) + (\hat{\Lambda}^{(1)T})_{\alpha\beta,ij}(\mathbf{r})\right)\times$$
$$\times(G^T)_{\alpha\varphi,il}(\mathbf{r},\mathbf{r}^{'},\tau) = \delta_{\beta\varphi}\cdot\delta_{jl}\cdot\delta(\mathbf{r}-\mathbf{r}^{'}), \quad \text{(S11)}$$

where

$$\hat{\Lambda}_{\alpha\beta,ij}^{(0)}(\mathbf{r},\nabla_{\mathbf{r}},\tau) = \delta_{\alpha\beta}\delta_{ij}\tilde{\tau} + \alpha H^2 h_\alpha h_\beta\delta_{ij} -$$
$$-i\eta H e_{\alpha\beta\gamma}h_\gamma\delta_{ij} - \xi_0^2\delta_{\alpha\beta}\delta_{ij}\Delta, \quad \text{(S12)}$$
$$\hat{\Lambda}_{\alpha\beta,ij}^{(1)}(\mathbf{r}) = -\delta V_{\alpha\beta,ij}(\mathbf{r}), \quad \text{(S13)}$$

symbol 'T' denotes matrix transpose by two pairs of indices, i.e. $(\Lambda^T)_{\alpha\beta,ij} = \Lambda_{\beta\alpha,ji}$. Using the definition of the Green's function, one can write the solution of equation (9) (from the main text of the article) as:

$$\chi_{\alpha\varphi,il}(\mathbf{r},\tau,H) = \int d^3r^{'}G_{\alpha\mu,im}(\mathbf{r}^{'},\mathbf{r},\tau)\times$$
$$\times\left(\delta V_{\varphi\mu,ml}(\mathbf{r}^{'}) - \langle V_{\gamma\mu,nm}\cdot\chi_{\gamma\varphi,nl}\rangle\right). \quad \text{(S14)}$$

Next, in order to calculate the correlater, entering the expression for χ, it is convenient to use the momentum representation. To do this let us introduce the Green's function in the momentum representation:

$$\mathcal{G}_{\alpha\beta,ij}(\mathbf{k},\mathbf{k}^{'},\tau) = \int d^3r d^3r^{'}e^{-i\mathbf{k}\mathbf{r}}G_{\alpha\beta,ij}(\mathbf{r},\mathbf{r}^{'},\tau)e^{i\mathbf{k}^{'}\mathbf{r}^{'}}, \quad \text{(S15)}$$

that satisfies the equation:

$$(\mathcal{G}^{(0)})_{\beta\alpha,ji}^{-1}(\mathbf{k},\tau)\mathcal{G}_{\varphi\alpha,li}(\mathbf{k},\mathbf{k}^{'},\tau) -$$
$$-\int\frac{d^3q}{(2\pi)^3}\delta\mathcal{V}_{\beta\alpha,ji}(\mathbf{k}-\mathbf{q})\mathcal{G}_{\varphi\alpha,li}(\mathbf{q},\mathbf{k}^{'},\tau) =$$
$$= (2\pi)^3\cdot\delta_{\beta\varphi}\cdot\delta_{jl}\cdot\delta(\mathbf{k}-\mathbf{k}^{'}), \quad \text{(S16)}$$

where $\mathcal{V}_{\beta\alpha,ji}(\mathbf{k})$ is the Fourier transform of the perturbation tensor, and the unperturbed Green function $\mathcal{G}_{\beta\alpha,ji}^{(0)}$ is, in its turn, the solution of the equation:

$$\left(\delta_{\alpha\varphi}\delta_{il}\cdot\tilde{\tau} + \alpha H^2 h_\alpha h_\varphi\delta_{il} - i\tilde{\eta}He_{\alpha\varphi\gamma}h_\gamma\delta_{il} + \right.$$
$$\left.\xi_0^2\delta_{\alpha\varphi}k^2\delta_{il}\right)^T(\mathcal{G}^{(0)T})_{\alpha\beta,ij}(\mathbf{k},\tau) = \delta_{\varphi\beta}\cdot\delta_{lj}. \quad \text{(S17)}$$

One can write the averaged over disorder Green's function in a form:

$$\langle\mathcal{G}\rangle_{\alpha\beta,ij}(\mathbf{k},\mathbf{k}^{'},\tau) = \delta(\mathbf{k}-\mathbf{k}^{'})\left[(\mathcal{G}^{(0)})_{\alpha\beta,ij}^{-1}(\mathbf{k},\tau) - \right.$$
$$\left. - \Sigma_{\alpha\beta,ij}(\mathbf{k},\tau)\right]^{-1}, \quad \text{(S18)}$$

where $\Sigma_{\alpha\beta,ij}(\mathbf{k},\tau)$ is a self-energy part, that can be represented through the sum over irreducible diagrams for the considered problem. It follows from the symmetry properties of the system that the tensor of self-energy part can be parametrised like:

$$\Sigma_{\alpha\beta,ij}(k=0,\tau_{\max}) = \left(\Sigma^{(1)}[\delta_{\alpha\beta} - \right.$$
$$\left.-h_\alpha h_\beta] - i\Sigma^{(2)}e_{\alpha\beta\gamma}h_\gamma + \Sigma^{(3)}h_\alpha h_\beta\right)\delta_{ij}, \quad \text{(S19)}$$

while the inverse tensor of averaged Green's function is expressed through the defined components of self-energy part in the following manner:

$$(\langle\mathcal{G}\rangle^{-1})_{\alpha\beta,ij}(k,\tau) = \left\{\tilde{\tau} - \Sigma^{(1)} + k^2\xi_0^2\right\}\delta_{\alpha\beta}\cdot\delta_{ij} - \\ -i\left\{\tilde{\eta}H - \Sigma^{(2)}\right\}e_{\varphi\beta\gamma}h_\gamma\cdot\delta_{ij} + \\ \left\{\alpha H^2 + \Sigma^{(1)} - \Sigma^{(3)}\right\}h_\beta h_\varphi\cdot\delta_{ij}. \quad \text{(S20)}$$

Note that in writing this expression, we neglected the renormalization of the coherence length associated with the dependence of the components $\Sigma^{(i)}$ on the wave vector. After substitution of the solution of the secular equation (19) (from the main text of the article), the Green's function matrix takes the same form as the self-energy part:

$$\langle\mathcal{G}\rangle_{\alpha\beta}(k,\tau_{\max}) = \mathcal{G}^{(1)}(\delta_{\alpha\beta} - h_\alpha h_\beta) + \\ +i\mathcal{G}^{(2)}e_{\alpha\beta\gamma}h_\gamma + \mathcal{G}^{(3)}h_\alpha h_\beta, \quad \text{(S21)}$$

where

$$\mathcal{G}^{(1)} = \frac{1}{2}\left(\frac{1}{\xi_0^2q^2} + \frac{1}{2\left|\tilde{\eta}H - \Sigma^{(2)}\right| + \xi_0^2q^2}\right), \quad \text{(S22)}$$

$$\mathcal{G}^{(2)} = \frac{1}{2}\left(\frac{1}{\xi_0^2q^2} - \frac{1}{2\left|\tilde{\eta}H - \Sigma^{(2)}\right| + \xi_0^2q^2}\right), \quad \text{(S23)}$$

$$\mathcal{G}^{(3)} = \frac{1}{\Sigma^{(1)} + \left|\tilde{\eta}H - \Sigma^{(2)}\right| - \Sigma^{(3)} + \alpha H^2 + \xi_0^2q^2}. \quad \text{(S24)}$$

3.

In order to analyze the properties of the phenomenological correlator of the effective impurity field $\Phi_{\alpha\beta\mu\nu}(\mathbf{q})$ introduced in the article, it is convenient to separate the functions $\Phi_i(\mathbf{q})$ into "structural" and "form" factors. We assume that the arrangement of impurities and the orientation of their spins are described by two functions: $C(r)$ – the pair correlation function of the aerogel and $m_s(r,\hat{s}^{(a)},\hat{s}^{(b)})$ – the spin pair correlation function in the ^{3}He film, where $\hat{s}^{(a,b)}$ is the unit vector in the direction of the averaged spin of a single impurity ("a" or "b" correspondingly). The spin pair correlation function can be represented as an expansion in powers of $(\hat{s}^{(a)}\cdot\hat{s}^{(b)})$, retaining the first three terms:

$$m_{ab}(r,\hat{s}^{(a)},\hat{s}^{(b)}) = m_s^{(0)} + m_s^{(1)}(r)(\hat{s}^{(a)}\cdot\hat{s}^{(b)}) + \\ +m_s^{(2)}(r)\left[(\hat{s}^{(a)}\cdot\hat{s}^{(b)})^2 - \frac{1}{3}\right], \quad \text{(S25)}$$

where $m_s^{(0)}$ is a constant, and $m_s^{(1,2)}(r)$ are functions that decay over the length of magnetic correlations. It is worth noting, that spin correlations are only possible under the condition of the ^{3}He film being single connected, covering the aerogel particles, i.e. the spin correlation implicitly depend on the correlations in the arrangement of impurities. Therefore, in order to account for this fact, it is convenient to consider a modified spatial correlation function from which the uncorrelated part $\tilde{C}(r) = C(r) - \frac{1}{V_0}$, is extracted, where V_0 – is the volume of the aerogel. Based on this, when averaging the correlator over disorder (after integration over the relative positions of two impurities $\mathbf{r}_{ab}$) three structural factors can be introduced according to the definition:

$$S_0(\mathbf{q}) = (N-1)\int d^3r e^{-i\mathbf{qr}}\cdot\tilde{C}(r), \quad \text{(S26)}$$

$$S_{1,2}(\mathbf{q}) = (N-1)\int d^3r e^{-i\mathbf{qr}}\cdot\tilde{C}(r)\cdot m_s^{(1,2)}(r), \quad \text{(S27)}$$

where $S_0(\mathbf{q})$ is the usual structural factor of the aerogel, related to the correlation in the arrangement of impurities, and $S_{1,2}(\mathbf{q})$ are magnetic structural factors, related both to the possible spatial inhomogeneity of the impurity distribution and to the magnetic correlations in the ^{3}He film. From the definition, the functions $S_{1,2}$ are zero if there are no spatial correlations in the arrangement of impurities or if the system lacks short-range order. Those parts of the function Φ_i that are obtained by averaging the product of the potentials of two impurities over the directions of their spins with the corresponding weights (powers of $(\hat{s}^{(a)}\cdot\hat{s}^{(b)})$ in the magnetic correlation function) will be denoted as $\varphi_i(\mathbf{q})$. In this formulation, they have the meaning of form-factors for different types of scattering. In total, four independent functions $\varphi_i(\mathbf{q})$ are sufficient to fully define the functions $\Phi_i(\mathbf{q})$. Then the expressions for Φ_i can be written as:

$$\Phi_0(\mathbf{q}) = n\cdot S_0(\mathbf{q})\cdot\varphi_0(\mathbf{q}) + n\cdot S_2(\mathbf{q})\cdot\tilde{\varphi}_2(\mathbf{q}), \quad \text{(S28)}$$

$$\Phi_{1,2}(\mathbf{q}) = n\cdot S_{1,2}(\mathbf{q})\cdot\varphi_{1,2}(\mathbf{q}), \quad \text{(S29)}$$

where $\varphi_0 \sim \sigma_{pp}^2$, $\varphi_1 \sim \sigma_{ep}^2$, $\tilde{\varphi}_2, \varphi_2 \sim \sigma_{ee}^2$. Let us note, that despite the fact that $\varphi_1 \gg \varphi_2$, the contribution to the considered effect from terms with Φ_2 may be more significant than from terms with Φ_1, since each of them is associated with its own type of magnetic correlations in the film through the functions $m_{1,2}(r)$. From the perspective of the correlator model discussed in Section 5 of the main text of the article, this difference is related to the difference in correlation lengths for these two terms, and the more significant contribution is the one with the longer correlation length.

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