Supplementary Material
to the article "Impact of critical spin fluctuations on the transport and magnetic properties of $EuFe_2As_2$ crystals"

I. CRITICAL BEHAVIOR OF MAGNETIC SYSTEM PARAMETERS IN THE VICINITY OF A PHASE TRANSITION

The concept of critical behavior, which groups various systems into universal classes, is effectively used to theoretically describe the behavior of physical quantities in the immediate vicinity of a continuous phase transition (in other words, a second-order phase transition) [1]. This concept assumes that critical indices belonging to the same universality class are identical, despite the fact that they are determined by changes in completely different order parameters. Within this concept, the critical behavior is determined by the static and dynamic critical indices.

A. Static critical indices

The critical behavior of thermodynamically equilibrium physical quantities such as heat capacity, magnetization, and magnetic susceptibility as the temperature changes near a phase transition is described by simple analytical expressions that differ from each other only in their critical indices (i.e., critical indices) [1, 2]:

$$C_h(T) = c_o \left(\frac{T - T_m}{T_m}\right)^{-\alpha}, \qquad T > T_m \tag{S1}$$

$$M(T) = m_o \left(\frac{T - T_m}{T_m}\right)^{\beta}, \qquad T < T_m \tag{S2}$$

$$\chi_0(T) = \chi_o \left(\frac{T - T_m}{T_m}\right)^{-\gamma}, \qquad T > T_m \tag{S3}$$

where $C_h(T)$ is the specific heat capacity measured in zero magnetic field, $M(T)$ is the magnetization, χ_0 is the magnetic susceptibility, c_o, m_o, χ_o are pre-exponential factors, T_m is the critical temperature, α, β, γ are the critical indices characterizing the specific heat capacity, magnetization, and magnetic susceptibility, respectively. We note that the derivative of the resistivity with respect to temperature $d\rho/dT$ changes with temperature according to the same law as the heat capacity (S1), with the same exponent α, with the difference only in the pre-exponential factors [3].

Static (time-averaged) critical indices depend only on the dimensionality of the system, the symmetry of the order parameter, and the interaction range [4], while remaining independent of many microscopic details. The entire diversity of phase transitions can be classified by their description methods (theoretical models) and universal classes, within which any processes are described in a universal manner. The values of the critical indices depend only on the order parameter dimension n and the system dimension d. This allows obtaining information about the order parameter of a system without detailed knowledge of its microscopic features. The static critical exponents for the main models, found from theoretical numerical calculations, are presented in Table S1. In addition to the critical exponents already mentioned, the table also lists those for the spin correlation function η and the correlation length ν.

Unlike the critical indices, the pre-exponential factors c_o, m_o, χ_o in above equations (1) - (3), which are responsible for the scale of physical quantities, can vary depending on the microscopic details of the system under consideration. To eliminate their influence, these expressions can be non-dimensionalized. For example, to remove the diamagnetic contribution to the magnetic susceptibility and thereby reduce the associated error, an alternative form of equation (3) can be used that does not include the pre-exponential factor:

$$\frac{\chi_0^{-1}(T)}{d\chi_0^{-1}(T)/dT} = \frac{T - T_m}{\gamma} \tag{S4}$$

TABLE S1. Static critical indices calculated by Monte Carlo simulations within the Heisenberg, Ising, and XY theoretical models for some spin n and structure d dimensions. ([1, 5]).

Model	n	d	α	γ	β	η	ν
3D-Heisenberg	3	3	-0.1336(15)	1.3960(9)	0.3689(3)	0.0375(5)	0.711(1)
3D-XY	2	3	-0.0146(8)	1.3177(5)	0.3485(2)	0.0380(4)	0.671(2)
3D-Ising	1	3	0.1099(7)	1.2372(4)	0.32648(18)	0.0364(4)	0.630(1)
2D-Ising	1	2	0	1.75	1.125	0.25	1

Similarly, for magnetic materials near the magnetic phase transition, the first derivative of the resistivity with respect to temperature is described by a formula similar to (S1) for heat capacity [6, 7]:

$$\frac{d\rho_s(T)}{dT} = \rho_{so}\left(\frac{T-T_m}{T_m}\right)^{-\alpha} + C \tag{S5}$$

where ρ_s is the contribution to the resistivity due to the interaction of charge carriers with critical spin fluctuations, $d\rho_s/dT$ is its derivative with respect to temperature, ρ_{so} is the pre-exponential factor, C is a constant, α is the critical index characterizing the specific heat capacity and the first derivative of the resistivity with respect to temperature, T_m is the critical temperature. This analogy is due to the fact that critical spin fluctuations make a significant contribution to the change in the resistance of iron pnictides in the vicinity of the magnetic ordering temperature [8].

B. Dynamic critical index

The dynamic critical index z characterizes the system's response to an alternating electromagnetic field. Therefore, to determine it, methods for resonant (electron spin resonance, and nuclear magnetic resonance), and non-resonant absorption of electromagnetic waves in the radio-frequency and microwave ranges are used.

A detailed analysis of critical behavior [9] showed that some dynamic parameters of systems belonging to the same static universality class may differ. This discrepancy is explained by the existence of different dynamic universality subclasses, which are described by dynamic scaling. The dynamic characteristics of a system depend not only on its dimensionality and the symmetry of the order parameter, but also on the laws of conservation of energy and spin, which impose certain constraints on scattering processes. This circumstance influences the magnitude of the correlation time τ_s (in our case, these are spin correlations), the index z appears in the description of the temperature dependence of which:

$$\tau_s(T) = \tau_0\left(\frac{\xi(T)}{\xi_0}\right)^{z} \tag{S6}$$

Here τ_0 specifies the time scale, being uniquely determined by the correlation length ξ. In turn, the correlation length, as the critical temperature approaches, diverges according to a power law with critical index ν:

$$\xi(T) = \xi_0\left(\frac{T-T_m}{T_m}\right)^{-\nu} \tag{S7}$$

where ξ_0 is the correlation length scale. Thus, the temperature dependence of τ_s is:

$$\tau_s(T) = \tau_0\left(\frac{T-T_m}{T_m}\right)^{-z\nu} \tag{S8}$$

As the critical temperature is approached, the intensity of the fluctuations increases: the correlation length increases, and the lifetime of the fluctuations becomes longer. Analysis of this process of critical slowing of fluctuations, i.e., the dependence $\tau_s(T)$, makes it possible to estimate the magnitude of the product $z\nu$.

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