Supplementary Material to the article
"Shot noise features of a terahertz superconducting bolometer"

The current-voltage characteristic of the bolometer superconducting film is determined by the stationary solution of the system of equations

$$\frac{dT_e}{dt} = -\frac{T_e - T_0}{\tau_{eff}} + \frac{I^2 R(T_e)}{c_e V} = 0, \tag{S1}$$

$$R(T_e) = \frac{R_0}{1 + \exp\{-(T_e - T_c)/\Delta T\}}, \tag{S2}$$

$$L\frac{dI}{dt} = U - I * R(T_e) = 0. \tag{S3}$$

Equation (S1) is obtained by eliminating the phonon subsystem temperature from equations (1,2) from the main text. For simplicity, we neglect the current dependence of the critical temperature here.

By differentiating equations (S1-3) with respect to voltage, it is easy to obtain an expression for the differential resistance:

$$\frac{dI}{dU} = \frac{1}{R(T_e)} \frac{1-\beta}{1+\beta}, \tag{S4}$$

where the parameter

$$\beta = \frac{T_e - T_0}{\Delta T}\left(1 - \frac{R(T_e)}{R_0}\right). \tag{S5}$$

The value of T_e in these expressions corresponds to the stationary solution of system (S1-3), that is, it is a function of the applied bias voltage U. It is easy to see that the region of the negative slope of the I–V characteristic corresponds to the condition $\beta > 1$.

Let us consider small deviations of the electron temperature and current from the stationary values: $T_e \to T_e + \delta T_e$; $I \to I + \delta I$. Linearizing equations (S1-3) with respect to these small deviations, we obtain:

$$\frac{d}{dt}\delta T_e = -\frac{\delta T_e}{\tau_{eff}} + \frac{T_e - T_0}{\tau_{eff}}\left\{2\frac{\delta I}{I} + \left(1 - \frac{R(T_e)}{R_0}\right)\frac{\delta T_e}{\Delta T}\right\}, \tag{S6}$$

$$\frac{d}{dt}\delta I = -\frac{R(T_e)}{L}\delta I - \frac{U}{L}\left(1 - \frac{R(T_e)}{R_0}\right)\frac{\delta T_e}{\Delta T}. \tag{S7}$$

In deriving these equations, equalities corresponding to stationary solutions (S1,3) were used, as well as an expression for the derivative

$$\frac{dR(T_e)}{dT_e} = \frac{R(T_e)}{\Delta T}\left(1 - \frac{R(T_e)}{R_0}\right). \tag{S8}$$

From equations (S6,7) it is easy to obtain expressions for the characteristic exponents of the system that determine its stability:

$$\lambda = \frac{1}{2\tau_{eff}} \left\{ \beta - 1 - \frac{\tau_{eff}}{\tau_L} \pm \sqrt{\left(\beta - 1 + \frac{\tau_{eff}}{\tau_L} \right)^2 - 8\beta \frac{\tau_{eff}}{\tau_L}} \right\}, \tag{S9}$$

where $\tau_L = L/R(T_e)$. The real part of at least one of the characteristic exponents can be positive only if $\beta > 1$, i.e., in the region of the negative slope of the I–V characteristic. The exact instability boundaries $\mathrm{Re}(\lambda) > 0$ depend on the ratio τ_{eff}/τ_L, which in our case is quite small.

Figure S1 shows the dependence of the real part of the larger characteristic exponent on the bias voltage U for various bath temperatures T_0. The graphs show that the instability region completely disappears only when the bath temperature is sufficiently close to the critical temperature $T_0 > 8.5\ K$. At the bias voltage $U = 0.6\ mV$ for which the main results of the article were obtained, the instability boundary lies just below the temperature $T_0 = 8.5\ K$. Using the full system (1-4) from the main text of the article will add a third characteristic exponent and slightly modify the result (S9), but will not qualitatively change the conclusions.

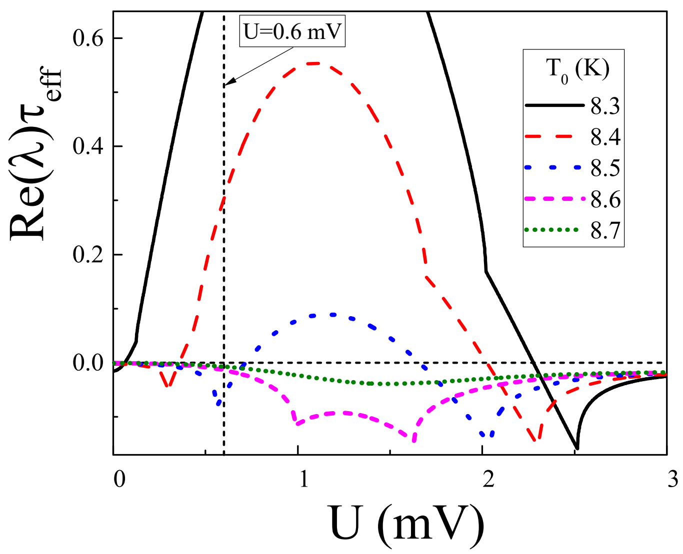

Fig.S1. Dependence of the real part of the characteristic exponent (S9) on the bias voltage at different values of bath temperature T_0.